

Dagstuhl-Seminar No. 01421

Integration of Algebra and Geometry Software Systems

October 15th to 19th, 2001

organized by

M. Joswig (TU Berlin) and

N. Takayama (Kobe University)

1 Summary

In many fields of modern mathematics specialized scientific software becomes increasingly important. Therefore, tremendous effort is taken by numerous groups all over the world to develop appropriate solutions. Topics include commutative algebra, D-modules, group and number theory, as well as algebraic, computational, discrete, and differential geometry.

With growing complexity of these software systems, the designers more than ever feel compelled to include functionality which might require techniques other than the roots of the respective system. Those techniques can often be found in other branches of mathematics. Instead of implementing new software individually it is preferable to make use of already existing stable software written by experts in their field. This raises the question for interfaces between software components. Several frameworks have been suggested and are still being discussed.

The seminar covered the following topics:

- Algorithms which require components from both geometry and algebra.
- Abilities and limitations of existing software systems from algebra and geometry.
- General purpose interfaces for mathematical software.
- Visualization components.
- XML based techniques for the presentation and the exchange of mathematical objects.

To ourselves, as the organizers, the undertaking of this workshop looked somewhat ambitious at the beginning. The participants came from quite diverse areas and we were a little afraid that there might be too little common ground. So we were relieved to see the participants enter very lively discussions in a friendly and open minded way from the first day. We are sure that the wonderful atmosphere of Schloss Dagstuhl helped a lot.

We hope that some this spirit will still be visible in a forthcoming book on the workshop's subject to be published by Springer.

As with probably all successful projects of a certain size it requires quite a few people to make it happen. We are grateful to all participants for contributing to this workshop. The staff of Schloss Dagstuhl, and, in particular, Reinhard Wilhelm generously offered a great opportunity. And, finally, Marc E. Pfetsch compiled the abstracts into this booklet.

Thanks to everyone,

Michael Joswig and Nobuki Takayama

2 Program

Monday, October 15th

- 9:00–9:10 Opening
- 9:10–9:40 James H. Davenport, University of Bath
Geometric Reasoning about the Algebra of Complex Functions
- 9:45–10:15 Hans Schönemann, Universität Kaiserslautern
Using External Tools with SINGULAR
- 10:20–10:50 David Saunders, University of Delaware
Smith Normal Form: 4 Algorithms and 2 access points
- 11:10–11:40 Jörg Rambau, Konrad-Zuse-Zentrum Berlin
TOPCOM: Making Use of Oriented Matroids in Discrete Geometry
- 11:45–12:15 Lutz Kettner, MPI Saarbrücken
CGAL, the Computational Geometry Algorithms Library
- 15:30–16:00 Konrad Polthier, TU Berlin
Differential Geometry Online
- 16:05–16:35 Steve Dugaro, Simon Fraser University
JVL, an Import/Export Tool for Maple Plots
- 16:40–17:10 Richard James Morris, University of Leeds
Web-based Client-Server System for the Calculation of Algebraic Surfaces
- 17:15–17:45 Hans-Christian Hege, Konrad-Zuse-Zentrum Berlin
Amira – a Modular and Extensible Software Environment for Virtual Laboratories

Tuesday, October 16th

- 9:00–9:30 John Abbott, University of Genova
New Features in CoCoA 5
- 9:35–10:05 Masayuki Noro, Kobe University
A Computer Algebra System Risa/Asir and OpenXM
- 10:10–10:40 Claude P. Bruter, Université Paris 12
Functional Applets
- 11:10–11:40 David Bremner, University of New Brunswick at Fredericton
*Matroid Polytope Completion:
Brute Force: Theme and Variations*
- 11:45–12:15 Michael Joswig, TU Berlin
Computational Convex Geometry with `polymake`
- 15:30–16:00 Jörg Rambau
Nominated talk: On the Cayley Trick

16:10–16:40 Konrad Polthier
Nominated talk: Persistent Digital Geometries

Wednesday, October 17th

- 9:00–9:30 Arjeh M. Cohen, TU of Eindhoven
OpenMath Tools
- 9:35–10:05 Ernesto Reinaldo Barreiro, TU of Eindhoven
A Tag Library for Developing Interactive Mathematical Documents
- 10:10–10:40 Marc Conrad, Southampton Institute
Object-oriented Design and Mathematics
- 11:10–11:40 Ulrich Kortenkamp, FU Berlin
Integrating CAS with Cinderella
- 11:45–12:15 Alfred Wassermann, Universität Bayreuth
GEONE_{GT} and its Connection to Computer Algebra Systems
- 14:00–18:00 Excursion

Thursday, October 18th

- 9:00–9:30 Jan Verschelde, University of Illinois - Chicago
Numerical Irreducible Decomposition using PHCpack
- 9:35–10:05 Nobuki Takayama, Kobe University
Generating Hypergeometric Function Identities by Gröbner Basis and Polyhedral Geometry
- 10:10–10:40 Ioannis Z. Emiris, INRIA - Sophia Antipolis
Combinatorial Geometry for Algebraic Elimination
- 11:10–11:40 Frank Lutz, TU Berlin and Konrad-Zuse-Zentrum Berlin
How to Recognize the Topological Type of a Manifold?
- 11:45–12:15 Ivan Dynnikov, Moscow State University
Knots in a Three-page Book
- 16:00–16:30 Komei Fukuda, ETH Zürich
Recent Progress in Polyhedral Computation
- 16:35–17:05 Marc Pfetsch, TU Berlin
Algorithmic Questions Connected to Polytopes
- 17:10–17:40 Oliver Labs, Universität Mainz
A Visual Introduction to Cubic Surfaces Using the Computer Software SPICY
- 19:30–20:00 Dongming Wang, Université Pierre et Marie Curie
Automated Generation of Diagrams with Maple and Java
- 20:05–20:35 Klaus Hildebrandt, TU Berlin
Integration of JavaView and webMathematica

20:40–21:10 Jean-Charles Faugère, CALFOR, LIP6, Paris
Interface between Computer Algebra Systems and Efficient Software

Friday, October 19th

9:00–9:30 Michael Joswig
Nominated talk: On the Polytope Completeness Problem

9:35–10:05 Ioannis Emiris
Nominated talk: Sparse Resultant Perturbations

10:40–11:10 Ulrich Kortenkamp
Nominated talk: Integrating CAS with Cinderella (da capo)

3 List of Participants

John Abbott, University of Genova
Ernesto Reinaldo Barreiro, TU of Eindhoven
Anna-Maria Bigatti, University of Genova
David Bremner, University of New Brunswick at Fredericton
Claude P. Bruter, Université Paris 12
Arjeh M. Cohen, TU of Eindhoven
Marc Conrad, Southampton Institute
James H. Davenport, University of Bath
Steve Dugaro, Simon Fraser University
Ivan Dynnikov, Moscow State University
Ioannis Z. Emiris, INRIA - Sophia Antipolis
Jean-Charles Faugère, CALFOR, LIP6, Paris
Komei Fukuda, ETH Zürich
Ewgenij Gawrilow, TU Berlin
Monika Gehweiler, Universität Tübingen
Hans-Christian Hege, Konrad-Zuse-Zentrum Berlin
Klaus Hildebrandt, TU Berlin
Michael Joswig, TU Berlin
Lutz Kettner, MPI Saarbrücken
Ulrich Kortenkamp, FU Berlin
Oliver Labs, Universität Mainz
Frank Lutz, TU Berlin and Konrad-Zuse-Zentrum Berlin
Richard James Morris, University of Leeds
Masayuki Noro, Kobe University
Martin Peters, Springer-Verlag – Heidelberg
Marc Pfetsch, TU Berlin
Konrad Polthier, TU Berlin
Eike Preuß, TU Berlin
Jörg Rambau, Konrad-Zuse-Zentrum Berlin
David Saunders, University of Delaware
Hans Schönemann, Universität Kaiserslautern
Nobuki Takayama, Kobe University
Jan Verschelde, University of Illinois - Chicago
Dongming Wang, Université Pierre et Marie Curie
Alfred Wassermann, Universität Bayreuth
Volkmar Welker, Universität Marburg

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1. Geometric Reasoning about the Algebra of Complex Functions

JAMES H. DAVENPORT (University of Bath)

On the one hand, we all “know” that $\sqrt{z^2} = z$, but on the other hand we know that this is false when $z = -1$. We all know that $\ln e^x = x$, and we all know that this is false when $x = 2\pi i$. How do we imbue a computer algebra system with this sort of “knowledge”? Why is it that $\sqrt{x}\sqrt{y} = \sqrt{xy}$ is false in general ($x = y = -1$), but $\sqrt{1-z}\sqrt{1+z} = \sqrt{1-z^2}$ is true everywhere?

It is the contention of this paper that, only by considering the geometry of \mathbb{C} (or \mathbb{C}^n if there are n variables) induced by the various branch cuts can we hope to answer these questions even semi-algorithmically. This poses questions for geometry, and calls out for a more efficient form of cylindrical algebraic decomposition.

2. Using External Tools with SINGULAR

HANS SCHÖNEMANN (Universität Kaiserslautern)

SINGULAR is a specialized CAS for polynomial computations. The kernel implements a variety of Gröbnerbase-type algorithms (generalized Buchbergers algorithm, standard basis in local rings, in rings with mixed order, syzygy computations, ...), algorithms to compute free resolutions of ideals, combinatorial algorithms for computations of invariants from standard bases (vector space dimensions, - bases, Hilbert function, ...) and algorithm for numerical solving of polynomial systems.

All others task have to use external tools, which include C/C++-libraries and external programs. Examples for both possibilities are given and our experience with them discussed:

- C/C++ libraries: fixed at build time, fast communication, difficult to change
 - GMP for multiple precision numbers
 - FACTORY for factorizing polynomials
- external programs, communication via ASCII encoded data: very flexible, relatively slow, easy to change
 - surf: visualization of curves and surfaces
 - (X)Emacs as front-end to SINGULAR
- external programs, communication via MP encoded data: not flexible, fast, easy to change
 - SINGULAR as a Gröbnerbase-engine within MuPAD

Every of these possibilities has its advantages and disadvantages – we have to provide interfaces to all of these types: this can be done via dynamic modules.

3. Smith Normal Form: 4 Algorithms and 2 access points

DAVID SAUNDERS (University of Delaware)

I will survey the literature on integer Smith normal form computation with an emphasis on four methods which hold interest for computation with large sparse matrices.

1. A particular form of elimination used by Heckenbach,
2. The “valence” algorithm of Dumas, Saunders, and Villard,
3. Giesbrecht’s gcd based method.
4. Eberly, Giesbrecht,’s solver based method.

The elimination method is one of many variants. It is rather successful in practice in a GAP package implementation. We will discuss some of the alternatives and considerations when using elimination.

The latter three methods are black-box methods. As a result they require very little memory, an important point for large sparse systems. Giesbrecht’s method is based on gcds of linear combinations of minors to obtain the determinantal divisors. The valence method first finds a restricted set of primes which may appear in the Smith form, then exploits the fact that the integer Smith form is the product of the p-adic Smith forms. The new algorithm of Eberly, et al. exploits a clever device to reveal invariant factors one by one thru linear system solving. It is fastest asymptotically and may be amenable to efficient practical implementation.

Two ways to access the implementations of elimination and the valence method in a GAP package will be described (web server and download) with some account of the joys and pitfalls of their creation. A LinBox library for exact sparse matrix computation is under development and will include Smith Form implementations in a way intended for easy use by other systems.

4. TOPCOM: Making Use of Oriented Matroids in Discrete Geometry

JÖRG RAMBAU (Konrad-Zuse-Zentrum Berlin)

TOPCOM is a package for computing triangulations. The input is a point configuration in affine space. A triangulation is a dissection of the convex hull of the configuration into simplices with vertices in the configuration such that any every two simplices intersect in a face of both.

In this talk, it is shown how oriented matroids serve as an interface between calculations in the coordinates of the input points and purely combinatorial computations in discrete geometry. The chirotope of the input configuration is a map that assigns to every affine basis the sign of the determinant (in homogeneous coordinates). This structure represents the complete combinatorial information, the “oriented matroid”, of the input configuration that is necessary to compute all triangulations of the input configuration. Elementary algorithms for the computation of the chirotope and the incorporation of symmetries into the enumeration of triangulations are explained.

5. CGAL, the Computational Geometry Algorithms Library

LUTZ KETTNER (MPI Saarbrücken)

CGAL is a C++ template library created in a European joint project, see www.cgal.org for the current release and further documentation. Its strength lies in correct and robust implementations based on exact arithmetic and degeneracy handling, in efficiency based on floating point filters and fast algorithms,

in flexibility based on templates and generic programming, and its excellent documentation. CGAL, as a library, provides interfaces to file formats for some of its data structures, and programming interfaces based on templates to utilize other libraries, e.g., exact arithmetic number types. It formalizes these programming interfaces in concepts similar to those known from the C++ Standard Template Library (STL).

6. Differential Geometry Online

KONRAD POLTHIER (TU Berlin)

The talk gives an overview of suitable geometry topics and of available multimedia tools to prepare online courses for students in differential geometry. The integrated online usage of software like Mathematica, Maple, and JavaView is demonstrated as well as new mathematical online services in geometry. Among our main efforts is the smooth integration of technology and classical education principles.

For further information we refer to the web site www.javaview.de, or to the new book “Multimedia Tools for Communicating Mathematics” which appeared at Springer Verlag.

7. Web-based Client-Server System for the Calculation of Algebraic Surfaces

RICHARD JAMES MORRIS (University of Leeds)

Algebraic surfaces, defined as the zero set of a polynomial function in three variables, present particular problem for visualizing, especially if the surface contains singularities. Most algorithms for constructing a polygonization of the surface will miss the singular points. We present an algorithm for polygonizing such surfaces which attempts to get accurate representations of the singular points. A client-server approach, with a java applet and a C program as backend, is used to enable the visualization of the polygonal mesh in a web browser. This system allows algebraic surfaces to be viewed in any web browser on any platform.

8. Amira – a Modular and Extensible Software Environment for Virtual Laboratories

HANS-CHRISTIAN HEGE (Konrad-Zuse-Zentrum Berlin)

Many scientific disciplines examine their objects of interest both experimentally and computationally. This concerns not only physics, chemistry, material sciences, geosciences and engineering, but also live sciences.

To facilitate the solution of the corresponding complex problems, suitable problem solving environments are needed – so-called “virtual labs”. These should offer all necessary aids (application-specific as well as generic ones) in an uniform coherent software environment. Furthermore they should provide visual support for all phases of the application work and put as little constraints on the users as possible, e.g. let them freely control the data flow, unconstrainedly combine the data, and interactively change the data, both manually and procedurally.

After stripping off application specific aspects, typically problems in the following fields arise: digital image analysis, geometry reconstruction, grid generation, shape analysis, solution of algebraic and differential systems, applied statistics, continuous and discrete optimization, as well as data visualization. From a mathematical point of view the tasks are mixtures of problems in computational geometry, computational topology, numerics, computational algebra, statistics, as well as digital image analysis and synthesis.

Amira, a software system developed at ZIB, provides an extensible framework for building such virtual labs, especially those with specific functionalities in 3D image analysis, grid generation and visual data analysis. In the talk, the software architecture of Amira is presented. Focusing on biomedical applications, typical tasks are explained and corresponding solutions in Amira are presented.

The author thanks all members of the Visualization Department of ZIB who contributed to this software large project.

9. New Features in CoCoA 5

JOHN ABBOTT (University of Genova)

This talk gives a brief overview of the CoCoA 5 project. The aims of the project are to produce both a library and an interactive system for performing computations in commutative algebra. Emphasis is on simplicity of use and high efficiency; these aims apply equally to the interactive system and to the open source C++ library. Furthermore, to facilitate interoperation with other programs CoCoA 5 includes a server accessible via an OpenMath-like interface. CoCoA 5 is a completely new project: CoCoA 4 has grown far beyond its humble beginnings which impose some inherent limitations. While still quite young, CoCoA 5 is already capable of computing Gröbner bases, and is significantly faster than its predecessor. Also, since CoCoA 4 began a number of new and improved algorithms have been discovered and which are being employed in CoCoA 5.

The facilities of C++ permit a sophisticated design which includes customized memory management, natural extensibility to new coefficient types, compact data representations, and a more mathematical interface. This convenience does not compromise efficiency significantly.

We note that the OpenMath standard is still in flux, and expect that our developments in CoCoA 5 will encourage crystallization of those parts relevant to computational commutative algebra. There is an active cooperation between CoCoA and OpenMath.

10. A Computer Algebra System Risa/Asir and OpenXM

MASAYUKI NORO (Kobe University)

OpenXM is an infrastructure for exchanging mathematical data. OpenXM also defines a client-server architecture for parallel and distributed computation. OpenXM package is a collection of clients and servers compliant to the OpenXM specification.

Risa/Asir is one of main components in OpenXM package. Risa/Asir is a software mainly for polynomial computation. Its major functions are polynomial factorization and Gröbner basis related computation. Its user language is C-like

and has gdb-like user program debugger. It has OpenXM interface as client and server and also provides library functions simulating connection to OpenXM asir server.

Goal of developing Risa/Asir is 1. to test new algorithms, 2. to be a center of a general purpose open system, 3. to provide environment for parallel distributed computation. Risa/Asir implements univariate and multivariate factorizer over various ground fields. As to Gröbner basis computation, Buchberger algorithm, F4 algorithm, change of ordering and rational univariate representation algorithm are implemented. Their performance are not excellent but not bad compared with other famous software such as CoCoA and SINGULAR.

It is a completely open system. It is an open source software and the OpenXM specification is completely documented. So, other software systems will be easily use the functions of Risa/Asir by implementing OpenXM client interface.

11. Functional Applets

CLAUDE P. BRUTER (Université Paris 12)

By functional applet is meant a frequently used subroutine used to compute or transform. Examples of functional applets are those which compute roots, functions, lines, triangulations, and so on. The scope of this naive contribution is to give a small list of such applets which seem to be useful both to the exhibition of usual and unusual topological and geometrical objects, and to show the impact of rational transformations in any vector space. For this last purpose, an other representation of vectors than the standard Cartesian one is used, which allows to extend the well known properties of the complex line to any vector space.

12. Matroid Polytope Completion: Brute Force: Theme and Variations

DAVID BREMNER (University of New Brunswick at Fredericton)

A (convex) polytope is the convex hull of a finite point set in \mathbb{R}^d . The *skeleton* of a polytope is formed by the vertices (0-faces) and edges (1-faces). The simplex method of linear programming walks from vertex to vertex along edges, improving the solution at each step. Hirsch conjectured almost 45 years ago that the diameter of the skeleton of a d dimensional polytope with n facets is at most $(n - d)$. Since there can be exponentially many vertices in terms of n and d , this conjecture is quite remarkable, although somewhat supported by the behaviour of the simplex method in practice. As of this writing, the the best bounds for the skeleton diameter are still superpolynomial.

Matroid polytopes are a purely combinatorial generalization of convex polytopes. If the Hirsch conjecture is false for convex polytopes, then it is also false for matroid polytopes. The validity of the Hirsch conjecture for matroid polytopes is also of independent interest. Perhaps most importantly, matroid polytopes provide a discrete search space that is relatively straightforward, if time-consuming, to search. With Klee, Babson, and Holt, I have begun a computer-search based investigation of the diameters of matroid polytopes. The general idea is to start with a long path, and search for a (matroid) polytope with this

path as the shortest path between two vertices. So far we have developed techniques to enumerate (i.e. generate) paths of a certain length and dimension, and I have written two programs to complete matroid polytopes from paths. The first of these programs has been successfully parallelized and runs with almost perfect speedup. Both of these programs represent rather simple approaches to searching based on backtracking. The main difference is the representation of the matroid polytope: in one case the well know chirotope is used; in the other the hyperline arrangement developed by Bokowski and colleagues is used. Since the search spaces involved are huge (nominally 2^{462} in the smallest case of interest), it is crucial to develop better pruning techniques. Part of the goal of this talk is to solicit suggestions for where techniques from e.g. optimization and computer algebra may profitably be applied.

13. Computational Convex Geometry with `polymake`

MICHAEL JOSWIG (TU Berlin)

`polymake` is a software package devoted to the study of convex polytopes. It primarily addresses researchers in polytope theory and related fields. The package has proved useful in a variety of research problems.

As a key feature `polymake` makes use of many successful implementations provided by other research groups. This way the system offers a unified interface to wide range of algorithms and tools.

The system's architecture is specifically designed for being extendible. Essentially there are two possibilities: Either via adding new interfaces (to be written in Perl) to already existing programs or via using `polymake`'s object oriented C++-library.

`polymake` has been developed jointly with Ewgenij Gawrilow.

14. Nominated talk: On the Cayley Trick

JÖRG RAMBAU (Konrad-Zuse-Zentrum Berlin)

The Cayley Trick is a one-to-one correspondence between mixed subdivisions of the Minkowski sum and subdivisions of the Cayley embedding of a set of given point configurations. This geometric relation appears also in sparse elimination theory.

In this talk, we introduce the basic geometric operations and explain the geometric correspondence. Moreover, we show a new interpretation in the framework of subdivisions induced by polytope projections. Applications to zonotopal tilings and triangulations of products of simplices are briefly discussed. (Joint work with Birkett Huber and Francisco Santos.)

15. Nominated talk: Persistent Digital Geometries

KONRAD POLTHIER (TU Berlin)

The talk discusses requirements for permanent storage of high-quality data sets, the expected benefits and the practical problems.

The EG-Models server at www.eg-models.de is introduced as a new electronic journal for the publication of digital geometry models from a broad range of mathematical topics. The geometry models are distinguished constructions,

counter examples, or results from elaborate computer experiments. Each submitted model has a self-contained textual description, is peer reviewed, and later reviewed by the “Zentralblatt für Mathematik”.

Talk concludes with an outlook on possibilities arising from such a collection of certified geometry models, especially, for the certification of algorithms and the smooth integration of software tools.

16. OpenMath Tools

ARJEH M. COHEN (TU of Eindhoven)

As a result of the OpenMath ESPRIT project, there is an OpenMath standard for the representation of mathematical objects, there are Content Dictionaries (approved by the OpenMath Society) for objects corresponding to MathML objects, and there are tools for manufacturing phrasebooks. We have explained that an OpenMath expression is a tree whose leaves can be four kinds of “constant”, variables, or symbols. The latter are defined in Content Dictionaries (CD). Each symbol is defined in a unique CD.

The official CDs can be found at <http://www.openmath.org>. But there are quite a few more. Some are experimental, but many others are meant for communication between a few applications, programs, or agents, and far from being as public as the official ones.

A phrasebook is understood to be the codec (taking care of translations from and to the application) together with the program that controls the communication. It is emphasized that the phrasebook needs to specify which CDs it uses and which actions it will perform on the OpenMath objects received.

There are three libraries for constructing phrasebooks. The one constructed at Eindhoven can be found at <http://crystal.win.tue.nl/download>, and contains several examples of existing phrasebooks, notably for Mathematica, COQ, and GAP. A brief discussion of the use of OpenMath for mathematical interaction on the Web, and requirements (such as a query language and brokerage) ended the talk.

17. A Tag Library for Developing Interactive Mathematical Documents

ERNESTO REINALDO BARREIRO (TU of Eindhoven)

In our talk we present a set of tools for the development of Mathematical Web Application. In particular, we focus our attention on a JSP tag library that easy the creation of interactive mathematical web pages. Among the features the tag library offers are: communication with backengines, like Mathematica and Gap, flow control to markup the internal logic of the page, context and session handling capabilities, etc. It is important to stress that our tools are OpenMath based (and oriented) and that has big advantages in terms of portability: the same web page could be either used, only small changes are necessary, with Mathematica or Gap as Backengine.

18. Object-oriented Design and Mathematics

MARC CONRAD (Southampton Institute)

Object oriented programming has well been established as a software design paradigm in the last years and is widely used in commercial software projects. However — in the context of mathematics it is rarely used, usually (if at all) in connection with a concrete programming language as C++ or Java. In contrast, we take here a language independent view, and show that there is a strong correspondence between mathematical structures and object oriented concepts. For instance, inheritance is similar to the specialization in mathematics (“*A field is a ring with ...*”), overriding of deferred methods coincides with the step from an abstract concept to a concrete example (“*Define x^2 as $x \cdot x$ and compute 3^2* ”), and polymorphic behavior corresponds with defining concepts on an abstract level (“*An elliptic curve over a field.*”).

The systematic assigning of responsibilities to objects is shown on the example of elliptic curves.

(Joint work with Susanne Schmitt, Saarbrücken)

19. Integrating CAS with Cinderella

ULRICH KORTENKAMP (FU Berlin)

After a short introduction of the dynamic geometry software package “Cinderella” (<http://www.cinderella.de>) and its mathematical kernel we discuss several possible interaction scenarios with other software. Typical interaction includes file exchange with other software (not necessarily geometry software), a plug-in architecture for user and 3rd party extensions and a way to replace internal functionality by alternative implementations. Two other items will be discussed in detail: using Cinderella as an input/output device by attaching its interface to other modules, and using Cinderella’s computation engine but using different visualization software. The first one is illustrated by a Mathematica notebook that drives a simulation visualized by Cinderella. This particular example introduces a very important concept that is both simple and powerful, and solves most software communication problems we encountered: Mathematica acts as a *user* of Cinderella (enabled to do so by a small API). This is to prevent inconsistencies that could arise in Cinderella out of wrong assumptions about the partnering software (here Mathematica). The last interaction scenario shows how Cinderella can talk to JavaView (<http://www.javaview.de>) using its automatic loci generation to display a surface corresponding to the realization space of a certain configuration.

20. GEONE_xT and its Connection to Computer Algebra Systems

ALFRED WASSERMANN (Universität Bayreuth)

GEONE_xT (www.geonext.de) is a free Java applet for interactive elementary geometry. Beside operations like the construction of standard elements like points, lines, circles and polygons and their interactions it can handle arbitrary algebraic expressions. This is enabled by a small Computer Algebra System

(www.hartmath.com) which makes it possible to plot functions, compute distances and angles and interlink functions with Euclidean Geometry elements. For more complex questions, which need the exact solving and simplification of polynomial systems we are developing an interface—written in Python—to the package “Geometry” of H.-G. Gräbe (Leipzig). This package is part of the SYMBOLIC DATA project (www.symbolicdata.org) and exists for various systems like Mupad, Maple, Reduce and Mathematica. The communication of GEONE_xT with other software is facilitated by the use of XML as file format for GEONE_xT constructions.

Another project in Bayreuth is DISCRETA, a program for the construction of combinatorial designs (www.mathe2.uni-bayreuth.de/~discreta). We intend to make the database of known parameter sets available on the WWW and make it possible to start the search for new parameter sets online.

21. Numerical Irreducible Decomposition using PHCpack

JAN VERSCHELDE (University of Illinois - Chicago)

Homotopy continuation methods have proven to be reliable and efficient to approximate all isolated solutions of a polynomial system. A recent development is the application of these methods to deal with components of solutions. In the developing field of numerical algebraic geometry we strive to develop numerically stable algorithms to compute an irreducible decomposition of the solution set of a polynomial system. This task involves elimination and factorization into irreducible components. We show how to use the software PHC in conjunction with Maple and how to call routines from PHCpack within a C program. This is ongoing joint work with Andrew Sommese (University of Notre Dame) and Charles Wampler (General Motors Research Laboratories).

22. Generating Hypergeometric Function Identities by Gröbner Basis and Polyhedral Geometry

NOBUKI TAKAYAMA (Kobe University)

This is an introductory survey on relations of hypergeometric functions, Gröbner bases, and convex polytopes and I explained my motivation of integrating algebra and geometry software systems. The following theorems opened a door to this interdisciplinary area:

1. There is a correspondence between the regular triangulations and sets of series solution basis of the GKZ hypergeometric system $H_A(\beta)$ for generic β . (Gel'fand, Kapranov, Zelevinsky, 1989)
2. The secondary cone of a regular triangulation T is the domain of convergence of the set of series solutions associated to T . (GKZ, 1989).
3. The secondary fan of A is refined by the Gröbner fan of the toric ideal I_A . (Sturmfels, 1990).

Upon these theorems and ideas of Hosono, Lian and S.T.Yau, we obtain a construction algorithm of series solution basis of $H_A(\beta)$ for any β . The main idea of the construction is the use of $\text{in}_{(-w,w)}(H_A(\beta))$ (SST, 2000).

Our OpenXM servers `ox_asir`, `ox_sm1`, `ox_tigers`, `ox_gnuplot`, `OpenMathproxy` (JavaClasses), `ox_mathematica`, `ox_m2`, (`ox_phc`) are collected to implement the

algorithm to construct series solutions (<http://www.openxm.org>). They will be used to generate, verify and present formulas for GKZ systems in a future in our project of electronic mathematical formula book.

23. Combinatorial Geometry for Algebraic Elimination

IOANNIS Z. EMIRIS (INRIA - Sophia Antipolis)

Eliminating variables from systems of algebraic equations is an important question in itself, as well as in order to numerically approximate all common zeros of the system. We focus on resultant methods for variable elimination. Such methods reduce system solving to a problem in linear algebra, namely an eigen-decomposition or a determinant factorization. The theory of toric (or sparse) elimination has been proposed in order to exploit the structure of the nonzero monomials in the input equations. It aspires to offer counterparts to all aspects of classical elimination theory, starting with the Newton polytope of a polynomial, which replaces the notion of total degree. Moreover, the mixed volume of Newton polytopes yields the generic number of common roots. It thus becomes apparent that several discrete geometric objects and operations are crucial in the context of toric elimination, including the construction of matrices expressing the toric resultant.

24. How to Recognize the Topological Type of a Manifold?

FRANK LUTZ (Technische Universität Berlin/ Konrad-Zuse-Zentrum Berlin)

It was shown by A.A. Markov in 1958 that the homeomorphism problem for manifolds is unsolvable in dimensions $d \geq 4$, i.e., there is no algorithm to decide whether two given manifolds M^d and N^d are homeomorphic or not. Still worse, S.P. Novikov proved that even the standard sphere S^d is not recognizable algorithmically when $d \geq 5$.

Despite of these results, there is a number of procedures and heuristics which, at least in special cases and situations, allow to recognize the topological type of particular manifolds. In this talk, we will survey such methods from a ‘practical point of view’.

25. Knots in a Three-page Book

IVAN DYNNIKOV (Moscow State University)

For any knot or link in the three-space, there exists an isotopic one that lies by whole in a three-page book, which is the union of three half-planes with common boundary, embedded in \mathbb{R}^3 . Such three-page knots have a simple combinatorial description which allows one to find many other three-page knots isotopic to the original one. By using this, a very fast partial algorithm for recognizing the unknot is constructed. In practice, it recognizes unknots given by a planar diagram with a few hundreds of crossings. A realization of the algorithm as a java applet and visualization by means of JavaView is in progress.

26. Recent Progress in Polyhedral Computation

KOMEI FUKUDA (ETH Zürich and ETFL Lausanne)

In this talk we look at the following three enumeration problems associated with convex polyhedra and present recent results.

1. Polyhedral Representation Conversion
 - Algorithms and Complexity
 - C/C++ Implementations
 - `cdd`, `cdd+`, `cddlib`
 - `lrs` by Avis
 - CPU Time
 - Closely related - polytope volume and redundancy removal
2. Constructing Arrangements and Zonotopes
 - Algorithms and Complexity
 - C Implementations – `rs_tope` using ZRAM
 - Application (with Liebling, Ferrez and Allemand)
3. Complete Listing of Arrangements and Configurations (with Finschi)
 - New Algorithms
 - Catalogs of Small Arrangements and Configurations

27. Algorithmic Questions Connected to Polytopes

MARC PFETSCH (TU Berlin)

In polytope theory one often comes across algorithmic questions. This for instance can happen, when trying to do experiments with polytopes on a computer. Several frameworks for using the computer as a tool for research appeared in the last years, e.g. `polymake` or `TOPCOM`. Also historically, the development of Linear Programming strongly increased the interest in algorithmic questions in polyhedral/polytope theory.

Therefore, it is interesting to investigate the algorithmic complexity of fundamental problems connected to polytopes. In this talk four examples of such problems are discussed:

1. Vertex Enumeration
2. The Steinitz Problem
3. Shellings of Simplicial Complexes
4. Isomorphism problems

Each of these problems highlights different aspects of such algorithmic problems. This talk presents joint work with Volker Kaibel, TU Berlin.

28. A Visual Introduction to Cubic Surfaces Using the Computer Software Spicy

OLIVER LABS (Universität Mainz)

At the end of the 19th century geometers like Clebsch, Klein and Rodenberg constructed plaster models in order to get a visual impression of their surfaces, that were so beautiful from an abstract point of view.

But these were static visualizations. Using computers it is now possible to draw algebraic curves and surfaces depending on parameters interactively by means of the program SPICY.

In this article we illustrate its usefulness by the famous construction of blowing-up the projective plane in six points (which gives a cubic surface in projective three-space).

When the user drags one of the six points, the equation and a raytraced image of the cubic surface is computed using external programs. As the whole process takes less than half a second, one nearly gets the impression of a continuously changing surface.

By means of this and Coble's explicit equations it was possible for the first time to visualize how some of the 27 lines upon the cubic surface meet, when it is deformed in such a way, that a double point occurs.

29. Automated Generation of Diagrams with Maple and Java

DONGMING WANG (Université Pierre et Marie Curie)

We show how to draw diagrams automatically from the predicate specification of a given set of geometric relations among a set of points in the plane. It is done first in Maple by translating the geometric relations into polynomial equations, decomposing the obtained system of polynomials into irreducible representative triangular sets, and finding an adequate numerical solution from each triangular set. A Java class coding the solution and the polynomials in each triangular set is generated, compiled, and then executed with the main Java programs to draw a diagram. The whole process combining symbolic elimination in Maple with numerical computation, graphic drawing, and letter labeling in Java is fully automatic. The drawn diagrams may be animated and fine-tuned by mouse click and dragging and saved as PostScript files. We present the drawing method, discuss some techniques of implementation, and give several sample diagrams drawn by our program, written as a function of GEOTHER (<http://calfor.lip6.fr/~wang/GEOTHER/>). A demonstration of the GEOTHER environment for geometric theorem proving is also included.

30. Integration of JavaView and webMathematica

KLAUS HILDEBRANDT (TU Berlin)

JavaView is a 3d geometry viewer and numerical software library written in Java. It allows to include 3D geometries in any HTML document, and to present interactive geometry experiments on the internet. JavaView also runs as an application from a Unix or Dos command prompt, and it can be attached as a 3D viewer to other programs.

JavaView's numerical software library is an open API with solutions and tools for problems in differential geometry and mathematical visualization. Its class library can be used and extended for own mathematical experiments in Java, while always profiting from the advanced 3D visualization capabilities and the web integration.

Additional information about JavaView and free download is available at <http://www.javaview.de/>.

webMathematica is a product of Wolfram Research released in October 2001. It connects Mathematica to the web. It is a server-based technology built on top of Java servlets. A webMathematica site can return content in many formats including HTML, various image formats, Mathematica notebooks, MathML, and TeX. It can work conveniently with many different web client technologies in browsers such as HTML forms, Java applets, JavaScript, Plug-ins, and ActiveX controls. webMathematica is also compatible with different server technologies such as servlets and Java Server Pages. webMathematica provides a collection of tools that allow Mathematica commands to be placed inside HTML pages; each time the page is requested from the server these commands are processed by a Mathematica session. In addition, the tools control the Mathematica sessions on the server and provide support such as launching, initialization, session pooling, and automatic restart.

During the testing of webMathematica we experimented with the smooth integration of webMathematica and JavaView, and generated a range of different server based mathematics applications shown at <http://www-sfb288.math.tu-berlin.de/~kah/webmat.htm>. In these examples we put a special focus on online mathematics visualization and tested how smooth interactive visualization and server-based computation can be integrated.

The simplest example uses JavaView as a 3D viewer for Mathematica graphics. One step further is to include panels for online modeling of the graphics. More complex applets are interactive interfaces to Mathematica solutions. Parameters of these solutions can be modified with sliders or by picking and dragging the points or vectors of the graphic in the viewer. Other examples combine functionality of JavaView and webMathematica, like adding textures to Mathematica graphics or clipping through geometries.

We worked out two ways for communication between JavaView and webMathematica: either to use the additional webMathematica command `MSPJavaView` or to communicate with the Mathematica server from a Java applet. The command `MSPJavaView[g]`, where `g` means any Mathematica graphics object, inserts a JavaView applet containing the computed graphics to the resulting HTML page. The usage of `MSPJavaView` does not require any knowledge in Java. It allows to add a variety of different JavaView applets to a Mathematica Server Page. The second type of communication with the Mathematica server is to write applets and use a URL connection. This equips the applet with the possibility to send requests to and to get responses of the Mathematica kernel. The JavaView library contains a special classes for the communication with a Mathematica server.

31. Nominated talk: On the Polytope Completeness Problem

MICHAEL JOSWIG (TU Berlin)

One of the notoriously open problems in computational geometry is the complexity status of the convex hull problem, that is, to compute a complete list of facets of a polytope defined as the convex hull of a given (finite) set of points. The question is whether or not there is an effective solution which requires polynomial time, measured in the sum of the sizes of input and output. While theoretically efficient algorithms are known for this problem in any fixed dimension, it is unclear whether such a method can be obtained with the dimension as part of the input.

It is known that the convex hull problem has a polynomial time reduction to the polytope verification problem: Given a finite set P of points and a finite set \mathcal{H} of affine halfspaces, decide whether $\text{conv}(P) = \bigcap \mathcal{H}$. By a straightforward transformation this is polynomial time equivalent to deciding whether the same holds for the special case where \mathcal{H} is a subset of the facet defining halfspaces of $\text{conv}(P)$. That is to say, to decide whether or not the set of facet defining halfspaces is complete.

We give a solution to this polytope completeness problem (for arbitrary polytopes) which relies on simplicial homology computation. This method, however, requires polynomial time only for (a slightly more general class than) simplicial polytopes. Up to a linear factor this agrees with the behavior of the best known algorithms for the convex hull problem.

It surprises that an algorithm which is of a very different flavor than all the known ones essentially has the same worst case time complexity.

This is joint work with Günter M. Ziegler.

32. Nominated talk: Sparse Resultant Perturbations

IOANNIS EMIRIS (INRIA - Sophia Antipolis)

We consider infinitesimal perturbations on sparse (or toric) resultants. This yields a general method for handling algebraic systems, even in the presence of “excess” components or other degenerate inputs. The complexity is simply exponential in the dimension and polynomial in the sparse resultant degree, thus capturing the polynomials’ structure by Newton polytopes and mixed volumes. The main tool are linear perturbations, easily computed by the combinatorial construction of sparse resultant matrices and compatible with the above notion of sparsity. Their application relies on the definition of a system with nonzero resultant. Our perturbation generalizes Canny’s Generalized Characteristic Polynomial (GCP) for the homogeneous case, while it provides a new and faster algorithm for computing Rojas’ toric perturbation. This work generalizes the well-known linear perturbation schemes, proposed in discrete and computational geometry for dealing with predicates on linear input objects. We examine the practical usefulness of our approach through its Maple implementation applied to specific examples.