# Dagstuhl Seminar 22221, Leibniz Zenter
# May 30 – June 03

## Day 1

| Monday 30 May 2022<br>**Mathematical theory** | |
|---|---|
| 08:45 – 09:00<br>09:00 – 10:00 | Welcome<br>Bernhard Beckermann |
| 10:00 – 10:30 | Coffee break |
| 10:30 – 11:00<br>11:00 – 12:00 | Ana Matos<br>Tomas Sauer |
| 12:15 – 13:45 | Lunch |
| 14:00 – 15:30 | Thematic discussion groups: Theory, Computation, Engineering, Bioscience |
| 15:30 – 16:00 | Coffee break |
| 16:00 – 17:30 | Thematic discussion groups: Theory, Computation, Engineering, Bioscience |
| 18:00 – 19:30 | Dinner |

## Day 2

| Tuesday 31 May 2022<br>**Numerical computation** | |
|---|---|
| 08:45 – 09:45<br>09:45 – 10:15 | Jürgen Gerhard<br>Gerlind Plonka |
| 10:15 – 10:45 | Coffee break |
| 10:45 – 11:45<br>11:45 – 12:15 | Hrushikesh Mhaskar<br>Daniel Potts |
| 12:15 – 13:45 | Lunch |
| 14:00 – 15:30 | Minicourse:<br>Connecting Maple with other Software |
| 15:30 – 16:00 | Coffee break |
| 16:00 – 17:30 | Free discussion |
| 18:00 – 19:30 | Dinner |

## Day 3

| Wednesday 01 June 2022<br>**Industrial applications** | |
|---|---|
| 08:45 – 09:45 | Ferre Knaepkens |
| 09:45 – 10:15 | Jürgen Prestin |
| 10:15 – 10:45 | Coffee break |
| 10:45 – 11:15 | Dirk de Villiers |
| 11:15 – 11:45 | Rina-Mari Weideman |
| 11:45 – 12:15 | Robert Beinert |
| 12:15 – 13:45 | Lunch |
| 14:00 – 15:30 | Free discussion |
| 15:30 – 16:00 | Coffee break |
| 16:00 – 17:30 | Free discussion |
| 18:00 – 19:30 | Dinner |

## Day 4

| Thursday 02 June 2022<br>**Life sciences** | |
|---|---|
| 08:45 – 09:45 | Johan Gielis |
| 09:45 – 10:45 | David Li |
| 10:45 – 11:15 | Coffee break |
| 11:15 – 12:15 | Richard Spencer |
| 12:15 – 13:45 | Lunch |
| 13:30 – 17:45 | Excursion: hiking |
| 18:00 – 19:30 | Dinner |

## Day 5

| Friday 03 June 2022<br>**Mathematical theory** | |
|---|---|
| 09:00 – 09:30 | Dmitry Batenkov |
| 09:30 – 10:00 | Hanna Veselovska |
| 10:00 – 10:30 | Coffee break |
| 10:30 – 11:00 | Mariya Ishteva |
| 11:00 – 12:00 | Check-out |
| 12:15 – 13:45 | Lunch |
| 14:00 – 18:00 | Departure |

# Limits of sparse super-resolution

***Dmitry Batenkov***
Department of Applied Mathematics, Tel-Aviv University, Israel

We consider the recovery of parameters $\{c_j, x_j\}$ in exponential sums

$$f(\omega) = \sum_{j=1}^{s} c_j \exp(\imath x_j \omega)$$

from bandlimited and noisy samples

$$f(\omega_i) + \epsilon_i, \qquad \omega_i \in [-\Omega, \Omega], \quad |\epsilon_i| \leq \varepsilon.$$

We discuss the conditioning of the problem when some of the exponents $\{x_i\}$ become close to each other, and show that all model parameters can be stably recovered provided that $\varepsilon \leq c(\Omega\Delta)^{2\ell-1}$, where $\ell$ is the maximal number of exponents which can be within an interval of size $\approx 1/\Omega$, and $\Delta$ is the a-priori minimal separation between the $\{x_j\}$.

We also discuss extensions of the analysis to generalized exponential sums $f(\omega) = \sum_{j=1}^{s}(a_j + b_j\omega)\exp(\imath x_j \omega)$, and connections to spectral properties of (confluent) Vandermonde matrices with nodes on the unit circle.

**References**

[1] D. Batenkov, L. Demanet, G. Goldman, and Y. Yomdin, "Conditioning of Partial Nonuniform Fourier Matrices with Clustered Nodes," *SIAM J. Matrix Anal. Appl.*, **44**(1), 199–220, 2020.
[2] D. Batenkov, B. Diederichs, G. Goldman, and Y. Yomdin, "The spectral properties of Vandermonde matrices with clustered nodes," *Linear Algebra and its Applications*, **609**, 37–72, 2021.
[3] D. Batenkov and G. Goldman, "Single-exponential bounds for the smallest singular value of Vandermonde matrices in the sub-Rayleigh regime," *Applied and Computational Harmonic Analysis*, **55**, 426–439, 2021.
[4] D. Batenkov, G. Goldman, and Y. Yomdin, "Super-resolution of near-colliding point sources," *Inf Inference*, **10**(2), 515–572, 2021.
[5] D. Batenkov and N. Diab, "Super-resolution of generalized spikes and spectra of confluent Vandermonde matrices," *arXiv:2203.11923*.

# Best rational approximants of Markov functions

***Bernhard Beckermann***
Department of Mathematics, Université de Lille, France

The study of the error of rational approximants of Markov functions

$$f^{[\mu]}(z) = \int \frac{d\mu(x)}{z-x}, \quad \mathrm{supp}(\mu) \subset [a,b],$$

on some $E \subset R \setminus [\alpha, \beta]$ has a long history, with a well-established link to orthogonal polynomials. For example, Zolotarev more than 100 years ago described best rational approximants and their error for the particular Markov function

$$f^{[\nu]}(z) = \frac{\sqrt{|a|}}{\sqrt{(z-a)(z-b)}} = \int \frac{d\nu(x)}{z-x}, \quad \frac{d\nu}{dx}(x) = \frac{\sqrt{|a|}}{\pi\sqrt{(x-a)(b-x)}},$$

for closed intervals $E$. The aim of this talk is to show that

$$\min_{r \in \mathcal{R}_{m-1,m}} \|1 - r/f^{[\mu]}\|_{L^\infty(E)} \leq 3 \min_{r \in \mathcal{R}_{m-1,m}} \|1 - r/f^{[\nu]}\|_{L^\infty(E)},$$

that is, up to some modest factor, the particular Markov function $f^{[\nu]}$ gives the worst relative error among all Markov functions $f^{[\mu]}$. In our proof we show similar inequalities for rational interpolants and Padé approximants.

**References**

[1] Bernhard Beckermann, Joanna Bisch, Robert Luce, Computing Markov functions of Toeplitz matrices, arXiv:2106.05098 (2021), appeared in **Numerical Algorithms**(2022).

# Total-Variation-Based Phase Retrieval in Optical Diffraction Tomography

***<u>Robert Beinert</u>*, *Michael Quellmalz***
Institute of Mathematics, TU Berlin, Germany

In optical diffraction tomography (ODT), the three-dimensional scattering potential of a microscopic object rotating around its center is recovered by a series of illuminations with coherent light. Reconstruction algorithms such as the filtered backpropagation require knowledge of the complex-valued wave at the measurement plane, whereas often only intensities, i.e., phaseless measurements, are available in practice.

In this talk, we propose a new reconstruction approach for ODT with unknown phase information based on three key ingredients. First, the light propagation is modeled using Born's approximation enabling us to use the Fourier diffraction theorem. Second, we stabilize the inversion of the non-uniform discrete Fourier transform via total variation regularization utilizing a primal-dual iteration, which also yields a novel numerical inversion formula for ODT with known phase. The third ingredient is a hybrid input-output scheme. We achieved convincing numerical results, which indicate that ODT with phaseless data is possible. The so-obtained 2D and 3D reconstructions are even comparable to the ones with known phase.

**References**

[1] R. Beinert and M. Quellmalz, Total variation-based phase retrieval for diffraction tomography arXiv:2201.11579 (2022).

# Chromatic Aberration in Large Antenna Systems

***Dirk de Villiers***
Department of Electrical and Electronic Engineering, Stellenbosch University, South Africa

It is well understood in electromagnetic field theory that harmonic plane waves, traveling in the same direction but emanating from different locations, interfere to cause a total field with a complex exponential frequency dependence. This effect often manifests in large (many wavelengths in size) antennas as a frequency ripple in the port reflection coefficient and radiation pattern magnitudes. The mechanism generating the ripple, or chromatic aberration [1], can normally be attributed to either multiple reflections from different structures in the antenna, or due to interference between direct and diffracted waves in the structure [2].

Often the resulting effect is small by virtue of the fundamental design of the structures at hand, but for certain applications even such small effects can influence the efficacy of the total system. Many radio astronomy science cases are examples of such sensitive applications, where the experiment searches for extremely feint signals (often deep in the noise) that need to be accurately characterized as a function of frequency. Here, we need to model our antenna system frequency responses very accurately in order to de-embed them from the data, and the chromatic aberration effects must be accounted for. Antennas typically used for radio astronomy, namely large reflectors or smaller antennas suspended over an artificial or natural ground plane, all suffer from this aberration to some degree. Previous efforts to model this effect are rather crude, and opens the door for more careful consideration and application of exponential analysis to reconstruct the required frequency responses without resorting to extremely time consuming high frequency resolution numerical analyses.

**References**

[1] D. Morris, Chromatism in radio telescopes due to blocking and feed scattering. Astronomy and Astrophysics 67, 221-–228, 1978.
[2] D.I.L. de Villiers, Prediction of Aperture Efficiency Ripple in Clear Aperture Offset Gregorian Antennas. IEEE Transactions on Antennas and Propagation 61(5), 2457-–2465, 2013.
[3] D.I.L. de Villiers, Wideband Reflector Antenna Beam Pattern Frequency Variation Modeling through the Characteristic Basis Function Pattern Method. In Int. Conference on Electromagnetics in Advanced Applications (ICEAA), 220–223., Cartagena de Indias, 2018.

# New Features in Maple 2022

***Jürgen Gerhard***
Research and Development, Maplesoft, Canada

An overview of the new features in Maple 2022 will be given, including formal power series, step-by-step solutions, intersection multiplicities, print layout mode, signal processing, and many more.

# Generalized Möbius-Listing bodies - new models for the sciences

***Johan Gielis***
Genicap Beheer, Tilburg, The Netherlands

Generalized Möbius-Listing surfaces and bodies (GML) were introduced by Ilia Tavkhelidze [1,2], building on the idea of Gaspard Monge to understand complex movements as the composition of simple movements. Another motivation is that the solutions of BVP for partial differential equations strongly depend on the topological properties of the domain in which the problem is considered.

A main focus of our joint research has been to classify all possible ways of cutting GML bodies (in analogy with the cutting of Möbius bands) [2,3]. In general the cutting process yields intertwined and linked bodies or surfaces of complex topology.

We defined the conditions under which the cutting process results in a single surface or body only, displaying the Möbius phenomenon of one-sidedness [4]. At the crossroads of geometry, topology, algebra and number theory, this gives rise to new ways of modeling and understanding certain dynamical processes in the natural sciences.

**References**

[1] I. Tavkhelidze, On the some properties of one class of geometrical figures. *Reports of Enlarged Sessions of the Seminar of I. Vekua Institute of Applied Mathematics* **16(1)**, 35-38, 2001
[2] I. Tavkhelidze, C. Cassisa, P.E. Ricci, About connection of generalized Möbius-Listing surfaces with sets of knots and links. *Lecture notes of Seminario Interdisciplinare di Matematica* **IX**, 183-200 2010.
[3] J. Gielis and I. Tavkhelidze, The general case of cutting of GML surfaces and bodies. *4OPEN* 3(7), 2020.
[4] J. Gielis, P.E. Ricci, I. Tavkhelidze, The Möbius phenomenon in Generalized Möbius-Listing surfaces and bodies, and Arnold's Cat phenomenon. *Advanced Studies: Euro-Tbilisi Mathematical Journal* **14**(4), 17-35, 2021.

# A modified Waring's problem: decoupling a multivariate polynomial

***Mariya Ishteva***[1]**, *Philippe Dreesen***[2]
[1] KU Leuven, Dept. Computer Science, ADVISE-NUMA, Belgium
[2] KU Leuven, Dept. Electrical Engineering, ESAT-STADIUS, Belgium

The Waring's problem for polynomials is a fundamental problem in mathematics, concerning the decomposition of a homogeneous multivariate polynomial $f(u_1, \ldots, u_m)$ of degree $d$ as

$$f(u_1, \ldots, u_m) = \sum_{i=1}^{r} w_i (v_1 u_1 + \cdots + v_m u_m)^d,$$

in which $r$ denotes the so-called Waring rank.

We consider a set of non-homogeneous polynomials and show how their joint decomposition can be computed using the canonical polyadic decomposition, which is a well-studied tensor decomposition. We also mention an application in nonlinear system identification.

**Reference**

[1] P. Dreesen, M. Ishteva, and J. Schoukens, Decoupling multivariate polynomials using first-order information and tensor decompositions. *SIAM Journal on Matrix Analysis and Applications (SIMAX)*, 36(2), 864–879, 2015.

# Least-squares Multidimensional Exponential Analysis

***Ferre Knaepkens***[1]***, Annie Cuyt***[1]***, Wen-shin Lee***[2]***, Yuan Hou***[1]
[1] Department of Computer Science, University of Antwerp, Belgium
[2] Computing Science and Mathematics, University of Stirling, United Kingdom

Exponential analysis consists in extracting the coefficients $\alpha_j$, $j = 1, \ldots, n$, and exponents $\phi_j$, $j = 1, \ldots, n$, of an exponential model

$$f(x) = \sum_{j=1}^{n} \alpha_j \exp(\phi_j x),$$

from a limited number of observations of the model's behaviour. Since there are $2n$ unkown parameters for $n$ exponential terms, this directly leads to a non-linear square system of $2n$ equations. However, in practice the signal is often perturbed by noise, hence, additional samples are collected and accumulated in an overdetermined non-linear system. Now the question remains how such a noisy overdetermined system behaves and how we can use this information to improve the accuracy of the results. In the case of a square system it is shown in [1] that the exponential analysis problem is deeply intertwined with Padé approximation theory and symmetric tensor decomposition, for both the one-dimensional and multi-dimensional cases. In particular the connection with Padé approximations is very interesting, since it allows the use of Froissart doublets to effectively filter out the noise and correctly estimate the number of terms $n$. It still remains to be shown that these properties also hold for the least-squares setting of the exponential analysis problem.

Furthermore, we focus on three different application domains, ranging from only one dimension to three-dimensional problems, each with its own challenges. First up is one-dimensional direction of arrival estimation, then image denoising of structured images and finally inverse synthetic aperture radar. We combine sub-Nyquist sampling, a validation technique based on Froissart doublets and new matrix pencil methods in order to tackle these challenging engineering applications.

**References**

[1] F. Knaepkens, A. Cuyt, and W.-s. Lee, From Exponential Analysis to Padé Approximation and Tensor Decomposition, in One and More Dimensions. In *Computer Algebra in Scientific Computing*, G. Vladimir, K. Wolfram, S. Werner, and V. Evgenii (eds.), 116–130. Lille, 2018.

# Advanced time-resolved imaging techniques and analysis and their applications in life sciences

***David Li***
Department of Biomedical Engineering, University of Strathclyde, Scotland, UK

Advanced time-resolved imaging techniques can reveal biological processes at the molecular level. They can unravel mechanisms underlying diseases and facilitate drug development. However, we also enter the low light regime (only a few photons are acquired), and new acquisition and analysis approaches are sought after.

# Solving a class of singular integral equations using rational approximation

***<u>Ana Matos</u>, Bernd Beckermann***
Laboratoire Paul Painlevé - Département de Mathématiques, Université de Lille ,France

We are interested in computing the unknown density of an equilibrium problem in logarithmic potential theory, where the support of the equilibrium measure is a finite union of distinct intervals. We will show that this problem is equivalent to solving a system of singular integral equations with Cauchy kernels. After fixing the functional spaces where we search for the solution , we obtain a theorem of existence and unicity. We then develop a general framework of a spectral method to compute an approximate solution , giving a complete error analysis. We will consider polynomial [1] and rational approximations [2,3], showing the advantage of using rational interpolation when the intervals are close. Inspired by the third Zolotareff problem, the poles and the interpolation points are chosen in such a way that we can ensure small errors. Some numerical examples showing the good approximation results will be given.

**References**

[1] A. Bultheel, P. Gonzalez Vera, J. Van Deun, On computing Rational Gauss-Chebyshev Quadrature Formulas, *Mathematics of Computation*, **Vol. 75**253, 307- 326, 2006.

[2] A. Bultheel, P. Gonzalez Vera, E. Hendriksen and O. Njastad, *Orthogonal rational functions*, Cambridge Monographs on Applied and Computational Mathematics, vol. 5, Cambridge University Press, 1999.

[3] S. Olver, Computation of equilibrium measures, *Journal of Approximation Theory* **163**, 1185-1207, 2011.

# Function approximation and machine learning

***H. N. Mhaskar***
Institute of Mathematical Sciences, Claremont Graduate University, U.S.A.

Two of the fundamental problems of machine learning are the following: (1) Given random samples from an unknown probability distribution, estimate the probability measure, (2) Given samples $\{(y_j, z_j)\}$ from an unknown probability distribution, estimate a functional relationship between $z_j$ and $y_j$. We explain why classical approximation theory as it was during our childhood is not adequate to solve the problem, and explain our efforts to use ideas of approximation theory to give a direct solution to the problem of estimating the functional relationship. We demonstrate that the problem of density estimation can be considered as a dual problem.

# From ESPRIT to ESPIRA: Estimation of Signal Parameters by Iterative Rational Approximation

***Gerlind Plonka**, Nadiia Derevianko, Markus Petz*
Institute for Numerical and Applied Mathematics, University of Göttingen, Germany

We consider exponential sums of the form

$$f(t) = \sum_{j=1}^{M} \gamma_j \, \mathrm{e}^{\phi_j t} = \sum_{j=1}^{M} \gamma_j \, z_j^t,$$

where $M \in \mathbb{N}$, $\gamma_j \in \mathbb{C} \setminus \{0\}$, and $z_j = \mathrm{e}^{\phi_j} \in \mathbb{C} \setminus \{0\}$ with $\phi_j \in \mathbb{C}$ are pairwise distinct. The recovery of such exponential sums from a finite set of possibly corrupted signal samples plays an important role in many signal processing applications, see e.g. in phase retrieval, signal approximation, sparse deconvolution in nondestructive testing, model reduction in system theory, direction of arrival estimation, exponential data fitting, or reconstruction of signals with finite rate of innovation.

Often, the exponential sums occur as Fourier transforms or higher order moments of discrete measures (or streams of Diracs) of the form $\sum_{j=1}^{M} \gamma_j \, \delta(\cdot - T_j)$ with $T_j \in \mathbb{R}$, which leads to the special case that $\phi_j = \mathrm{i}T_j$ is purely complex, i.e., $|z_j| = 1$.

We introduce a new method for **E**stimation of **S**ignal **P**arameters based on **I**terative **R**ational **A**pproximation (ESPIRA) for sparse exponential sums. Our algorithm uses the AAA algorithm for rational approximation of the discrete Fourier transform of the given equidistant signal values. We show that ESPIRA can be interpreted as a matrix pencil method applied to Loewner matrices. These Loewner matrices are closely connected with the Hankel matrices which are usually employed for recovery of sparse exponential sums. Due to the construction of the Loewner matrices via an adaptive selection of index sets, the matrix pencil method is stabilized. ESPIRA achieves similar recovery results for exact data as ESPRIT and the matrix pencil method (MPM) but with less computational effort. Moreover, ESPIRA strongly outperforms ESPRIT and MPM for noisy data and for signal approximation by short exponential sums.

**References**

[1] N. Derevianko, G. Plonka, and M. Petz, *From ESPRIT to ESPIRA: estimation of signal parameters by iterative rational approximation.* IMA Journal of Numerical Analysis, to appear.

# Interpretable Approximation of High-Dimensional Data based on ANOVA Decomposition

***Daniel Potts**, **Michael Schmischke***
Faculty of Mathematics, Technische Universität Chemnitz, Germany

We consider fast Fourier based methods for the approximation of high-dimensional multivariate functions. Our aim is to learn the support of the Fourier coefficients in the frequency domain, where only function values of scattered data available. Based on the fast Fourier transform for nonequispaced data (NFFT) we will use the ANOVA (analysis of variance) decomposition in combination with the sensitivity analysis in order to obtain interpretable results. We will couple truncated ANOVA decompositions with the NFFT and compare the new method to other approaches on publicly available benchmark datasets.

**References**

[1] D. Potts and M. Schmischke, Approximation of high-dimensional periodic functions with Fourier-based methods. *SIAM J. Numer. Anal.* 59, 2393-2429, arXiv:1907.11412, 2021
[2] D. Potts and M. Schmischke, Interpretable Approximation of High-Dimensional Data. *SIAM J. Math. Data. Sci.* 3, 1301-1323, arXiv:2103.13787, 2021
[3] F. Bartel, D. Potts, and M. Schmischke, Grouped Transformations in High-Dimensional Explainable ANOVA Approximation. SIAM J. Sci. Comput., arXiv:2010.10199 2022

# Detection of directional higher order jump discontinuities with shearlets

***Jürgen Prestin***[1]***, Kevin Schober***[1]***, Serhii Stasyuk***[2]
[1] Institute of Mathematics, University of Lübeck, Germany
[2] Institute of Mathematics, National Academy of Sciences of Ukraine, Kyiv, Ukraine

In this talk we consider trigonometric polynomial shearlets which are special cases of directional de la Vallée Poussin type wavelets to detect singularities along curves of periodic bivariate functions. This generalises the one-dimensional case discussed in [1], where singularities occur in single points only.

Here we provide sharp lower and upper estimates for the magnitude of inner products of the shearlets with the given function. The size of these shearlet coefficients depends not only on the distance to the curve singularity, but also on the direction of the singularity.

In the proofs we use orientation-dependent localization properties of trigonometric polynomial shearlets in the time and frequency domain, cf. [1]. We also discuss jump discontinuities in higher order directional derivatives along edges, see [2].

**References**

[1] H. N. Mhaskar and J. Prestin, On the detection of singularities of a periodic function. *Adv. Comput. Math.* **12**(2-3), 95–131, 2000.
[2] K. Schober, J. Prestin and S. A. Stasyuk, Edge detection with trigonometric polynomial shearlets. *Adv. Comput. Math.* **47**(1), Paper No. 17, 41 p., 2021.
[3] K. Schober and J. Prestin, Analysis of directional higher order jump discontinuities with trigonometric shearlets. *Math. Found. Comput.* Online First, 2021.

# Multi-exponential Functions and Continued Fractions

***Tomas Sauer***[1,2]
[1]Chair of Mathematical Image Processing & FORWISS, University of Passau, Germany
[2]Fraunhofer Institute for Integrated Circuits, Department X-ray Development Centar, Passau, Germany

The recovery of multi-exponential functions of the form

$$f(x) = \sum_{k=1}^{r} f_k \, \rho_k^x,$$

from integer samples is closely related to the theory of continued fractions. The connection is, among others, made by the Hankel matrices

$$H_n := \big( f(j+k) : j,k = 0,\dots,n \big)$$

and the rational best approximants to the Laurent series

$$\lambda(x) = \sum_{k=0}^{\infty} f(k) \, x^{-k-1}, \qquad x \neq 0.$$

The talk highlights some of these connections and also discusses some possibilities for extensions to several variables.

**References**

[1] T. Sauer, *Continued Fractions and Signal Processing.* Springer, 2021.

# An Inverse Problems Framework for Magnetic Resonance Relaxometry and Macromolecular Mapping

***Richard G. Spencer***
National Institute on Aging, National Institutes of Health, Baltimore, Maryland, USA

The success of conventional magnetic resonance imaging (MRI) is partly attributable to the fact that it is a Fourier technique. Data is collected in the space reciprocal to spatial coordinates, known as k-space, and then (inverse) Fourier transformed to produce an image. This reconstruction has the very attractive property of being mathematically well-conditioned, with condition number of unity, so that noise in the acquired data is necessarily transmitted to the image domain but is not magnified. Because of this, early studies performed at low magnetic field and with relatively unsophisticated radio frequency technology were able to yield useful images. Correspondingly, the quality of conventional MRI has increased roughly in proportion to improvements in acquisition SNR. However, the newer technique of MR relaxometry is not so fortunate; the reconstruction of acquired data to obtain parameter distribution functions is via a version of the inverse Laplace transform. This arises from the classically ill-posed problem of inverting the Fredholm equation of the first kind. One implication is that noise is amplified in the reconstruction process, so that brute force efforts to improve SNR rapidly reach the point of diminishing returns, and other means must be undertaken to produce useful results. For this, the inverse problems perspective has proven to be enormously fruitful. We will discuss some aspects of this formalism applicable to MR relaxometry and related experiments. Our main application is to macromolecular mapping, particularly myelin mapping in the brain, and we will show how more accurate myelin quantification permits physiological correlations to be established. Our studies have the twofold goal of improving the capacity of MR to diagnose pathology and monitor disease progression, and of developing methods of general use for inverse problems.

# Super-Resolution on the Two-Dimensional Unit Sphere

***Anna Veselovska***
Technical University of Munich, Department of Mathematics and Munich Data Science Institute

In this talk, we discuss the problem of recovering an atomic measure on the unit 2-sphere $\mathbb{S}^2$ given finitely many moments with respect to spherical harmonics. Our analysis relies on the formulation of this problem as an optimization problem on the space of bounded Borel measures on $\mathbb{S}^2$ as it was considered by Y. de Castro & F. Gamboa [1] and E. Candés & C. Fernandez-Granda [2]. We construct a dual certificate using a kernel given in an explicit form and make a concrete analysis of the interpolation problem, which provides theoretical guarantees for the recovery of atomic measure on $\mathbb{S}^2$. Next to that, we analyze such a problem from a numerical perspective in an extensive series of experiments, using a semidefinite formulation of the optimization problem and its discretized counterpart.

This is a joint work with Frank Filbir and Kristof Schröder.

**References**

[1] De Castro, Y., & Gamboa, F., Exact reconstruction using Beurling minimal extrapolation. *Journal of Mathematical Analysis and applications*, **395**(1), 336–354, 2012.
[2] Candés E.J., Fernandez-Granda C. , Super-Resolution from Noisy Data. *J. Fourier Anal. Appl.*, **19**, 1229–1254, 2013.
[3] Filbir, F., Schröder, K., & Veselovska, A., Recovery of Atomic Measures on the Unit Sphere. *Numerical Functional Analysis and Optimization*, 1–41, 2022.

# Antenna Position Estimation through Sub-Sampled Exponential Analysis of Signals in the Near-Field

***Rina-Mari Weideman*[1], *Ridalise Louw*[1], *Ferre Knaepkens*[2], *Dirk de Villiers*[1], *Annie Cuyt*[2], *Wen-Shin Lee*[3], *Stefan J. Wijnholds*[1,4]**

[1] Dept of Electrical & Electronic Engineering, Stellenbosch University, South Africa
[2] Dept of Computer Science, University of Antwerp, Belgium
[3] Division of Computing Science and Mathematics, University of Stirling, Scotland, UK
[4] Dept of Innovation & Systems, Netherlands Institute for Radio Astronomy (ASTRON), The Netherlands

Antenna position estimation is an important problem in large irregular arrays where the positions might not be known very accurately from the start. We present a method using harmonically related signals transmitted from an Unmanned Aerial Vehicle (UAV), with the added advantage that the UAV can be in the near-field of the receiving antenna array. The received signal samples at a chosen reference antenna element are compared to those at every other element in the array in order to find its position. We show that the method delivers excellent results using ideal synthetic data with added noise. Furthermore, we also simulate the problem in a full-wave solver. Although the results are less accurate than when synthetic data are used, due to the effects of mutual coupling, the method still performs well, with errors smaller than 4% of the smallest transmitted wavelength. Finally, we show how our method can detect whether the cables of two antennas were accidentally switched, and how a simple mutual coupling calibration method can improve the results even further.