Multiple-valued Logic

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(editors)

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This Dagstuhl Seminar brought together approximately 60 researchers covering the full spectrum of the current research on many-valued logics, ranging from mathematical foundations to computational issues and applications. Several young researchers could attend this meeting, and have fruitful interactions with more established researchers from Japan, South and North America, and from an exceptionally large number of European countries. Their contributions may be classified under the following four headings.

Enhancement of the theoretical basis. Despite its long tradition, work on foundational issues in many-valued logics still spawns new and thrilling problems and results. The rich and intriguing properties of the algebraic counterparts of many-valued logic should be firstly mentioned in this context. This algebraic perspective was strongly represented in this seminar. In the same spirit, albeit in a different setting, several talks dealt with fuzzy set theory, and their relations with rule based control.

Automated deduction: theory and tools. As a focal point in this field one must also mention the current research on complete and sound proof systems – in particular for infinitely-valued logics. Two notable examples are given by Łukasiewicz logic and by product logic. As amply discussed during the Dagstuhl seminar, the calculi corresponding to these logics require far reaching generalizations of the classical techniques arising from Hilbert-style systems, Gentzen systems and resolution. A recurring theme in this field is the use of formulas with regular sets of signs.

*Compiled by Bernhard Beckert, University of Karlsruhe.

Modelling and reasoning with incomplete and uncertain knowledge. Many-valued calculi are often encountered, for instance, in possibility theory and possibilistic logic. They provide a natural framework for manipulating preference criteria between various interpretations. They also find use in clarifying many entrenched situations in non-monotonic reasoning. In this context we should also mention such topics as the logical manipulation of uncertainty factors, which were the main concern of a number of talks in the Dagstuhl meeting.

Applications. Applications of many-valued logics – both in the infinitely-valued and in the finitely-valued case – are an important source of motivation for future research. During this seminar important talks focused on applications in hardware design and verification, unit commitment in power systems, and in biomedical engineering.

This Seminar was also a workshop/conference in the COST Action #15 on "Many-valued Logics for Computer Science Applications", the only existing European Action on Computer Science issues. One of the objectives of this Action was the dissemination of knowledge accumulated and produced within the project to the worldwide scientific community *outside* the COST action itself.

This objective has been very successfully accomplished with this Dagstuhl seminar.

Daniele Mundici
Peter H. Schmitt

Abstracts of the Talks

In the order in which the talks were given.

Walter Carnielli, Campinas (Brazil)

Non-deterministic Semantics

This talk introduces a new form of combining semantics, with the double intention of, first, to offer alternative semantic interpretations to certain less understood logic systems, and second, to combine simple logics so as to obtain other logics with richer structure.

Let L be a propositional language. By a *non-deterministic semantic framework for* L we mean a pair $ND = \langle T, M \rangle$ where T is a collection of transformations (called *translations*) from L to a family of languages $L_{\lambda \in \Lambda}$ governed by a set of *axioms* or *conditions*, and $M = \{M_{\lambda \in \Lambda}\}$ is a class of *basic models*, where L_λ and M_λ have the same type of similarity of L.

It is possible then to define a *non-deterministic forcing relation* based on ND, generalizing from the conceptual framework of possible-worlds semantics (viewing the transformations in ND as abstractions of possible worlds, and the conditions on translations in ND as abstractions of accessibility relations).

We discuss three example-cases of non-deterministic semantics, stressing the role of many-valued matrices as basic models: the first one, showing how to associate multi-valued semantics to logics having non-truth-functional connectives (as in the paraconsistent logics), the second showing how to give interpretations to certain many-valued logics in terms of classes of logics having lower number of truth-values, and the third showing how to combine Kripke models to form new logics.

Although the investigated examples involve just a finite set of basic models in M, it is very natural to extend these definitions in terms of pre-sheaves of structures, with interesting consequences. In particular, a challenging problem is to define appropriate algebraic operations reflecting this construction, which would obtain new algebraic counterparts for the logics involved.

Matthias Baaz, Vienna (Austria)

Analytic Calculi for Many-valued Logics

This lecture describes the impact of proof theoretic investigations on many-valued logics using three examples:

1. The first-order variant of Avron's Hypersequent Calculus for infinitely valued Gödel logic makes it possible to gain a clear understanding of the properties of the completeness proof of Takeuti-Titani.

2. The identification of fragments of Hajek's Basic Logic BL with Urquat's logic C and its extensions by residuation makes it easy to analyze derivations in BL and to demonstrate the independence of the "commutativity

of the minimum"-axiom from the other axioms of *BL*. A further proof-theoretic analysis of *BL* leads to an analytic formalization and consequently to decidability.

3. The possibility to deal with infinitary calculi/derivations in an effective way is connected to quantifier elimination of the underlying metatheory.

Hiroakira Ono, Ishikawa (Japan)
Many-valued Logics as Logics without the Contraction Rule

Logics lacking some or all of structural rules, when they are formulated in sequent calculi, are called *substructural logics*. The class of substructural logics includes Lambek calculus, logics without the contraction rule (BCK logics), linear logic, relevant logics and so on. The study of substructural logics will enable us to discuss these different kinds of logics within a uniform framework. Now, it is expected to be one of the central topics in non-classical logic.

Here, we are trying to develop a general theory of logics without the contraction rule, i.e., extensions of the intuitionistic logic without the contraction rule, $\mathbf{FL}_{\mathrm{EW}}$ (see e.g. [1,2] for the details). Our main tool is to use algebras which are related to the logic $\mathbf{FL}_{\mathrm{EW}}$, which are known as *residuated lattices*. First, we will develop a basic algebraic study of residuated lattices and will show that models of Łukasiewicz's many-valued logics and the model of product logic (from fuzzy set theory) come out naturally from their algebraic properties.

Then we will discuss *neighbours* of the classical logic over $\mathbf{FL}_{\mathrm{EW}}$. By applying Jónsson's Lemma, we can give a criterion for a logic to be a neighbour of the classical logic. As a consequence, we can show that there are infinitely many neighbours of the classical logic, each of which is different from any of Łukasiewicz $m+1$-valued logic with a prime number m.

By discussing many-valued logics in such a broader context, we will be able to give a clearer view of them. Moreover, this approach will contribute to building a bridge between the study of substructural logics and that of many valued logics and fuzzy logic. For, though many of the results on the former logics are apparently related to the latter, as we have sketched in the above, they are sometimes unknown in the study of the latter logics. The whole contents of our study will appear in our forthcoming paper "Logics without the contraction rule and residuated lattices".

References

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[2] H. Ono: Semantics for substructural logics. In K. Došen and P. Schroeder-Heister (eds.): *Substructural Logic*, pp. 259–291, Clarendon Press, Oxford, 1993.

Reiner Hähnle, Karlsruhe (Germany)
Proof Theory of Many-valued Logic and Hardware Design

We show that tableau and sequent rules for many-valued logics are closely related to many-valued decision diagrams and generalized formula decompositions as used in logic design and hardware verification.

Although some tools and methods used in linear optimization, automated theorem proving, logic design and hardware verification are common, distinct notation is used and there seems to be a negligible amount of communication among these communities. We endavour to show where parallels occur, but also where differing points of view lead to a different emphasis, thereby establishing a basic concordance.

Gonzalo Escalada-Imaz, Barcelona (Spain)
Determining the Truth Degree in Real Time Settings

Until the beginning of this decade, controlling real-world industrial processes was done by *classical control theory.* However, currently it is widely accepted that whenever the mathematical model of the process is complex, *Classical control theory* fails to design a suitable controller.

Given this fact, and the capital relevance of controlling industrial processes, a fragment of the AI research community has turned its attention to this promising field. The goal consists in designing KBS-based controllers to work within the strong conditions that have to be met in *real time settings* (RTS), imposed by the dynamic evolution process.

Controllers working in RTS must remain continuously active because each time the process state changes, a control signal must be produced in a bounded time (reactivity property). Also, some kind of *temporal* and *approximated reasoning* is needed in order to follow the inherently dynamic nature of the process and to deal with inexact information.

Our approach *approximating reasoning* relies on *regular multiple-valued logic* (RML) which enables to capture vague, imprecise and partial information. Thus, our KBS is based on RML rules enriched pertinently with a *temporal* structure to reason about time and so, to follow the dynamic changes of the process.

Finally, the underlaying general model of the controller is an AND-connector tree where with each node a truth degree is associated. The truth degree of a father AND-connector is obtained by a function whose arguments are the truth degrees of the AND-connector's sibling nodes.

A compiler running off-line obtains a table data structure tied to each AND-connector. These tables have the crucial feature that, for each configuration of the truth values modeling the current state of the process, the truth value degree of each father AND-connector is computed in $O(1)$ time from the child nodes. This performance together with the $O(1)$ complexity of the bottom-up propagation of truth values ensures the *reactivity* condition.

As a final remark, we add that the proposed controller has been inspired by a real-world application framework, a pediatric intensive care unit.

Peter Vojtáš, Košice (Slovakia)
Many-valued Logic Programming and Abduction

In our talk we present declarative and procedural semantics of many-valued definite logic programming (MVDLP) and prove its completeness.

In order to fit real world data and multiple agent behaviour, our connectives are arbitrary and subject of learning/approximation. First decision is that our program consists of implications of form

$$A \;\leftarrow\; \vee\;([\&_1(B_1, \&_2(B_2, B_3))], [\&_3(C_1, \&_4(C_2, C_3))]).cf = a \in [0,1]$$

So we skip the clausal notation in DNF (because in arbitrary many-valued logics the formula $\neg B \vee A$ needs not to be equivalent to $B \to A$; for another approach see work of Mundici). Different $\&_i$'s in the body correspond to different nature of datas, depending, e.g., on different user environments and/or stereotypes. $\vee$ in the body serves to aggregate single findings to the global confidence. Note that rule is equipped with a confidence factor, hence our programs are fuzzy theories in the sense of Pavelka, Novák and Hájek.

The deduction is based on backward usage of many-valued modus ponens

$$\frac{(B,x),(B \to A, y)}{(A, \mathcal{C}_{\to}(x,y))}$$

Namely a query A? using the above rule develops into the query

$$\mathcal{C}_{\to}\,((a, \vee\;([\&_1(B_1, \&_2(B_2, B_3))], [\&_3(C_1, \&_4(C_2, C_3))]))?$$

The only conditions we have to assume are Pedrycz-Gottwald conditions $\Phi2(\mathcal{C}_{\to}, \to)$ and $\Phi3(\mathcal{C}_{\to}, \to)$. We prove completeness of our semantics according to all left continuous connections (both t-operators), their finite approximations (without associativity) and aggregation (compensatory) are the operators of Zimmermann. We also show that minimal solutions of many valued definite abduction restricts to composition of MVDLP and a linear programming problem. Relevant TEX files are at `http://leibniz.math.fu-berlin.de/~vojtas`.

Rasim Egri, Ankara (Turkey)
with Ismet Erkmen
A New Fuzzy Approach to Unit Commitment in Power Systems

In this study, a new approach to solve Unit Commitment, one of the basic problems in power systems, using fuzzy set theory is proposed. Unit Commitment procedure provides the most economic and feasible schedules of "on" and "off" time periods for the generating units in a power system for the near future. A generation schedule is feasible if the schedule successfully meets various operational constraints of the system. However, due to the uncertainties coming from the nature of the problem, like the unknown demand and the unit production cost, an exact analytic solution is difficult to obtain. Therefore these uncertainties are modeled using fuzzy logic in our approach. The performance and the sensitivity of the proposed technique are tested on a sample power system; the results are compared with the ones that are optained by the well established Dynamic Programming method. It is observed that the performance and the robustness of the proposed method is as good as the Dynamic Programming method. Considering the linguistic simplicity and the computational efficiency of the proposed technique, it can be said that it is a potential alternative for solving the Unit Commitment problem in power systems. The present extension to this study is on the tuning of the fuzzy logic controller and the application of the new technique to a real power system, namely the Turkish Power System.

Hans Jürgen Ohlbach, London (UK)
with Jana Koehler, Freiburg (Germany)
How to Augment a Logical System with a Boolean Algebra Component

We investigate how to augment a given logical system, for example an arithmetical equation solver, with a Boolean component. The *atomic decomposition* technique proposed in this talk reduces reasoning about the Boolean component in the combined system to reasoning in the pure basic system only.

A typical instance of this scheme is a linear programming system which is to be augmented with reasoning about cardinalities of sets, or other functions mapping sets to integers. The sets and their set-theoretic relationships are axiomatized with propositional logic. Atomic decomposition then reduces reasoning about numerical attributes of these sets to arithmetic equation solving.

References

[1] H. J. Ohlbach and J. Koehler: How to Augment a Logical System with a Boolean Algebra Component. To appear in W. Bibel and P. H. Schmitt (eds.): *Automated*

Deduction – A Basis for Applications, volume III, Kluwer, Dordrecht.

Christian Fermüller, Vienna (Austria)
Finite-valued Logics and Classical Proof Theory

We report on joint work of the *VGML* (Vienna Group in Many-valued Logics) consisting of M. Baaz, C. G. Fermüller, G. Salzer, and R. Zach. Finitely-valued logics are used as a tool to illuminate classic concepts of proof theory. Claiming that there exists a systematic relation between two concepts like the classical sequent calculus **LK** and natural deduction for classical logic, is a void statement as long as one does not consider broad classes of concepts of which the ones to be compared are just particular instances. We use the family of all finitely-valued logics with arbitrary truth functional connectives and distribution quantifiers as basis to substantiate the claim that important concepts of Gentzen-style proof theory can be "derived" from each other. In particular we consider

- the relation between many-placed sequents and multi-conclusional natural deduction systems,
- truth tables and operators for reducing cuts with corresponding types of cut-formulas, and
- the relation between the well-known syntactical restriction that turns **LK** into **LJ**, on the one hand, and Kripke semantics for logics of intuitionistic type, on the other hand.

Tools like signed resolution allow to automatize the various translations of concepts. The system MULTLOG demonstrates this strikingly: Given any finite set of truth tables as input MULTLOG outputs various types of calculi along with soundness and completeness proofs as a LaTeX documents.

Agata Ciabattoni, Bologna (Italy)
with Dov Gabbay, London (UK), and Nicola Olivetti, Torino (Italy)
Cut-free Proof Systems for Logics of Weak Excluded Middle

In this talk we explore logics which arise from well known systems by weakening the excluded-middle principle. In particular, we introduce cut-free hypersequent calculi for the LQ logic, obtained by adding to Intuitionistic Logic the weak law of the excluded middle, that is $\neg A \vee \neg\neg A$, and for systems, called W_n, obtained by adding to affine Linear Logic (without exponential connectives) the n-weak law of the excluded middle, that is $\neg A \vee (A \oplus \cdots \oplus A)$ ($n-1$ times). For $n = 3$, the system W_n coincides with 3-valued Łukasiewicz Logic; for $n > 3$, it is a proper subsystem of n-valued Łukasiewicz Logic. Then, our calculi can be seen as a step forward in order to find hypersequent calculi, in which the cut-elimination theorem holds, for finitely-valued Łukasiewicz Logics.

Ewa Orlowska, Warsaw (Poland)
with Mihir Chakraborty, Calcutta (India)

Many-valued Substitutivity Principles

We consider two families of many-valued logics: logics whose many-valuedness is numerical and logics with non-numerical many-valuedness. These classes of logics include, among others, many-valued information logics, Rosser-Turquette logic, fuzzy logic. For several logics from each of the two groups we propose a semantics for the identity predicate and we prove validity of the underlying substitutivity principles. We also discuss substitutivity principles that hold for p-compatible identities in algebraic theories and substitutivity principles in E-unification theory.

Felip Manyà, Lleida (Spain)
with Ramón Béjar, Lleida (Spain), and Gonzalo Escalada-Imaz, Barcelona (Spain)

Solving the SAT-problem in Regular CNF-formulas

First of all, we present a Davis-Putnam-style satisfiability checking procedure for regular propositional formulas in conjunctive normal form (CNF-formulas). For the sake of efficiency, we have equipped the procedure with a suitable data structure for representing formulas. This data structure allows, for example, to detect the existence of unit clauses in constant time and to apply the regular unit clause rule with a linear-time worst-case complexity.

Second, we define several regular branching rules: An optimized version of Hänhle's branching rule, a regular version of the positive clause rule and a branching rule based on the concept of maximal set of truth values.

Third, we describe a generator of random k-SAT instances of the satisfiability problem in regular CNF-formulas.

Fourth, we show the experimental results obtained after executing an implementation of the algorithm on a distribution family of randomly generated regular 3-SAT instances: It turns out that for 3 truth values and 60 propositional variables, near the ratio $C/V = 6.17$ – where C and V are, respectively, the number of clauses and propositional variables – we find the hardest instances. The number of nodes of the proof tree generated by the procedure increases exponentially near this ratio and quickly decreases beyond than. For 5 truth values and 60 propositional variables, we observe the same effects near the ratio $C/V = 8.17$.

Finally, we show that the 2-satisfiability problem in regular CNF-formulas can be solved in polynomial time using a refinement of the proof procedure proposed.

Siegfried Weber, Mainz (Germany)

Conditional Objects Based on MV-algebras

Abstract not available.

Ulrich Höhle, Wuppertal (Germany)

Singletons and Fuzzy Partitions

Let $M = (L, \leq, *)$ be a GL-monoid [1, Section 5] satisfying the additional distributivity law

$$\alpha * (\bigwedge_{i \in I} \beta_i) = \bigwedge_{i \in I} (\alpha * \beta_i) \qquad \text{for all } \alpha \in L \text{ and all } \{\beta_i \mid i \in I\} \subseteq L \ .$$

Typical examples are complete Heyting algebras or complete MV-algebras. The M-valued interpretation of the formalized theory of identity and existence [2, Subsection 3.2] leads to the following concept of MV-valued sets (X, E) where X is a non-empty set and $E : X \times X \longmapsto L$ is a map (so-called *M-valued equality on X*) subjected to the following axioms

$$\text{(E1)} \quad E(x,y) \leq E(x,x) \wedge E(y,y) \qquad (\textit{Strictness})$$

$$\text{(E2)} \quad E(x,y) = E(y,x) \qquad (\textit{Symmetry})$$

$$\text{(E3)} \quad E(x,y) * (E(y,y) \to E(y,z)) \leq E(x,z) \qquad (\textit{Transitivity})$$

A subset $\{f_i \mid i \in I\}$ of L^X is called an *L-fuzzy partition* of the universe X iff $\{f_i \mid i \in I\}$ satisfies the following *disjointness condition*:

$$\bigvee_{x \in X} ((\bigvee_{\ell \in I} f_\ell(x)) \to f_i(x)) * f_j(x) \quad \leq$$
$$[\mathbf{E}(f_i) * (\bigwedge_{y \in X} f_i(y) \to f_j(y))] \ \wedge \ [\mathbf{E}(f_j) * (\bigwedge_{y \in X} f_j(y) \to f_i(y))]$$

where $\mathbf{E}(g) = \vee\{g(x) \mid x \in X\}$. Then the following theorem holds [2, Theorem 4.2.2]: A subset $\mathcal{Z} = \{f_i \mid \in I\}$ of L^X is an L-fuzzy partition of X iff there exists an M-valued equality E on X such that

- Every map $f_i \in \mathcal{Z}$ is a singleton of (X, E),
- $E(x,x) \quad = \quad \bigvee_{i \in I} f_i(x) \qquad$ for all $x \in X$.

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[2] U. Höhle: On the fundamentals of fuzzy set theory. *J. Math. Anal. Appl.*, 201:786–826, 1996.

Antonio Di Nola, Naples (Italy)
with Ada Lettieri

One Chain Generated Varieties of MV-algebras

MV-algebras constitute a generalization of Boolean Algebras and arise from the many-valued logic of Łukasiewicz in the same manner as boolean algebras arise from two-valued logics.

An MV-algebra is an algebraic structure $A = (A, \oplus, *, 0)$ such that $(A, \oplus, 0)$ is an abelian monoid and the following identities hold: $x^{**} = x$; $x \oplus 0^* = 0^*$; $(x^* \oplus y)^* \oplus y = (y^* \oplus x)^* \oplus x$.

If we set $x \otimes y = (x^* \oplus y^*)^*$, $x \wedge y = (x \oplus y^*) \otimes y$, and $x \vee y = (x \otimes y^*) \oplus y$, for every $x, y \in A$, then $(A, \vee, \wedge, 0, 1)$ is a bounded distributive lattice, which is called the reduct of A and denoted by $L(A)$. Boolean algebras coincide with MV-algebras satisfying the additional identity $x \oplus x = x$. Let A be an MV-algebra. The set $B(A) = \{x \in A \mid x \oplus x = x\}$ is a boolean algebra. Actually it is the greatest boolean subalgebra of A.

The variety $\vartriangle \Rrightarrow$ of all MV-algebras coincides with the variety HSP[0,1] generated by the MV-algebra defined on the real unit $[0, 1]$ as follows:

$$x \oplus y = \min(1, x + y), \quad x \otimes y = \max(0, x + y - 1), \quad x^* = 1 - x \ .$$

The main results presented here, are the following:

1. A characterization of the sub-varieties satisfying the amalgamation property; actually, they are those that have exactly one generator.

2. There exists a categorical equivalence between the sub-variety generated by a finite chain MV-algebra with n elements and the category whose objects are pairs (B, R_n) where $B = (B, +, \cdot, -, 0, 1)$ is a boolean algebra and $R_n \subseteq B^n$ such that:

   i) If $(b_0, b_1, \ldots, b_{n-1}) \in R_n$, then $b_0 \geq b_1 \geq \cdots \geq b_{n-1}$;

   ii) If $(b_0, b_1, \ldots, b_{n-1}) \in R_n$, then $(\overline{b_{n-1}}, \overline{b_{n-2}}, \ldots, \overline{b_0}) \in R_n$;

iii) If $(b_0, b_1, \ldots, b_{n-1}),\ (c_0, c_1, \ldots, c_{n-1}) \in R_n$, then

$$\begin{aligned}(b_0 &+ c_0, \\ &\vdots \\ b_k &+ c_k + \textstyle\sum_{i+j=k-1} b_i \cdot c_j, \\ &\vdots \\ b_{n-1} &+ c_{n-1} + \textstyle\sum_{i+j=n-2} b_i \cdot c_j)\end{aligned}$$

is an element of R_n;

iv) $(b, b, \ldots, b) \in R_n$ for all $b \in B$.

3. Every relation R_n can be represented by a boolean algebra B and a vector of ideals of B.

4. A characterization of the automorphisms of $B(A)$ which can be extended to automorphisms of A.

5. Following I. R. Goodman, H. T. Nguyen, and E. A. Walker, we introduce the notion of *Abstract Conditional MV-Space*, as an MV-algebra having the reduct lattice to be an Abstract Conditional Space. These seem to be the suitable algebraic structures coping with the notion of conditional events in the framework of Łukasiewicz logic. Indeed we provide a characterization of such a class of MV-algebras.

6. Finally we characterize the group of automorphisms of an n-valued algebra.

Zbigniew Stachniak, Toronto (Canada)
From Inferentially and Referentially Finitely-valued Systems to Resolution

Are there many-valued logics or just logics with many-valued semantics? Is the semantic apparatus of many truth-values a distinctive feature of many-valued logic or is it the intended interpretation of an applied language, a philosophical commitment, that necessitates the choice of multiple-valued interpretation for the language? These issues seem to be still unresolved in spite of almost 80 years of continuous research in the area of many-valued logics.

In the maze of opinions concerning the defining features for the class of many-valued logics, there is still one path that can be explored. The bivalent nature of classical logic is clearly represented in the theses of this calculus. Other logical systems (such as Leśniewski's Protothetic or Ontology) are formalized in such a way as to make the intended interpretation of an applied language evident from the start, at the proof theoretic level.

In this talk we are concerned with the search for a proof-theoretical evidence of finite-valuedness within the class of cumulative inference systems. We define and investigate the notion of an inferentially many-valued inference system. In this definition we try to capture the idea of the proof-theoretical representation of truth-functional many-valued semantics. We contrast the notion of inferential many-valuedness with semantic (or referential) many-valuedness.

Although the notions of inferential and referential many-valuedness do not coincide, standard (i.e., structural and compact) inferentially or referentially finitely-valued logics are resolution logic in the sense of [1]. Hence, every such system $\mathcal{P}$ has a resolution counterpart, i.e., there exists a deductive proof system based on the (non-clausal) resolution principle which is refutationally equivalent to $\mathcal{P}$.

References

[1] Z. Stachniak: *Resolution Proof Systems: An Algebraic Theory.* Kluwer Academic Publishers, 1996.

Dirk Van Heule and Albert Hoogewijs, Gent (Belgium)

The Partial Predicate Calculus. A Three-valued Object Logic for the Theorem Prover Isabelle

In this talk we define the 3-valued Partial Predicate Calculus (PPC) as an object logic for the generic theorem prover Isabelle. We will focus on the propositional logic. It will be necessary to add some new definitions and rules to the semantics of PPC in order to use the proving mechanisms of Isabelle in natural deduction style.

The truth-tables for the logical operators *negation* ($\neg$) and *conjunction* ($\wedge$) correspond to Kleene's notation. The logical operators are extended with a *non-monotone* operator Δ to express the *definedness* ($\Delta\alpha \equiv T$) or undefinedness ($\Delta\alpha \equiv F$) of a formula.

	$\neg$	Δ
T	F	T
F	T	T
U	U	F

$\wedge$	T	F	U
T	T	F	U
F	F	F	F
U	U	F	U

The definition of *validity* and *consequence* differs from most other 3-valued logics. In PPC, a valuation is a mapping $\mathcal{V} : \text{Form}_{PPC} \mapsto \{T, F, U\}$ with Form_{PPC} being the well-formed formulas of PPC. $\mathcal{M}$ is a *model* for a set of formulas Σ iff for all $\gamma \in \Sigma$: $\mathcal{V}(\gamma) \equiv T$. A formula α is *valid* for a model $\mathcal{M}$ ($\mathcal{M} \models \alpha$) iff $\mathcal{V}(\alpha) \in \{T, U\}$. A formula is valid iff it is valid for all models of PPC. A formula

α is a *consequence* of a set of formulas Σ ($\Sigma \models \alpha$) iff for all models $\mathcal{M}$ of Σ, $\mathcal{M} \models \alpha$.

By choosing Isabelle as theorem proving engine for PPC, we had to transform PPC into a natural deduction calculus, PPC_nat. Because the "assumption calculus" of PPC is not really a sequent calculus nor a natural deduction calculus, we had to reorganize, redefine and modify most of the rules. Also we had to add new rules.

Due to the definition of validity and consequence in PPC, we had to express the difference between a formula A at the left-hand side and a formula B at the right-hand side of $\vdash$, but also the difference between $A \vdash B$ and $\dfrac{\Gamma \vdash A}{\Gamma \vdash B}$. This was done by introducing a new connective "!", where $!A \equiv (A \wedge \Delta A)$ means "A is a true formula (trueF)".

In this talk we discuss the translation of the inference rules of PPC into those of PPC_nat. We show some results including the use of the cut-rule and the modification of the modus ponens.

Erik Rosenthal, New Haven (USA)
A Linear Resolution Rule for Annotated Logics

Signed logics provide a classical logic framework for reasoning about multiple-valued logics. In particular, signed resolution restricted to regular sets – upsets or complements of upsets – unifies the resolution and reduction rules of annotated logics. However, with these inference rules, the linear restriction, which is desirable in a variety of settings, including logic programming, precludes restriction to regular signs. A hyperresolution-like extension that preserves regularity is available, but linearity is artificial since, in effect, the reduction steps are "hidden."

The inference rule $\curlywedge$-*resolution*, currently under development, may be regarded as a replacement for annotated resolution/reduction. If the domain of truth values is linear, reduction is never necessary, and $\curlywedge$-resolution is identical to annotated resolution. Otherwise, $\curlywedge$-resolution admits linear proofs, and preliminary analysis indicates that it is more efficient than either signed resolution or annotated resolution/reduction with regard to both proof space and search space.

Petr Hájek, Prague (Czech Republic)
Takeuti-Titani Logic Revisited

In my forthcoming book [1], I pay much attention to the study of three fuzzy logics (both propositional and predicate) given by the well known most important continuous t-norms (Łukasiewicz, Gödel, product). The t-norm defines the semantics of conjunction &, its residuum is the truth function of implication $\rightarrow$; various other connectives are definable. Analyzing the approach of Takeuti and Titani [2] I develop (in Chap. IX, Sect. 1 of [1]) a predicate logic having three different conjunctions (as above), three corresponding implications and two corresponding negations as well as rational truth constants (as in Pavelka logic). This logic has obvious semantics and is not recursively axiomatizable (since Łukasiewicz predicate logic over $[0,1]$ is not). I present an axiomatization whose axioms are those of the three logics for the respective connectives, finitely many axiom schemes for truth constants and for connectives from different logics. Deduction rules are modus ponens, (double) generalization and an infinitary rule whose simplest case reads:

$$\frac{\varphi \to \psi \veebar \bar{r} \text{ for all } r > 0}{\varphi \to \psi}$$

Here $\veebar$ is strong Łukasiewicz disjunction. Completeness of the logic is proved.

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Marc Roubens, Liege (Belgium)

Some Basic Fuzzy Set Algebraic Operations Revisted

The most elementary operations for usual sets, as the union and the intersection of any two sets and the complement of any set, can be generalized for the same operations on fuzzy sets with the use of a de Morgan triple (T, S, n) where T is a t-norm and S the associated t-conorm defined with the strict negation n.

It is well known that these extensions give impossibility results if one wants the idempotence property and the contradiction law to hold simultaneously.

Pexiderized extensions of distributivity properties and the classical boolean relation $(A \cap B) \cup (A \cap B^C) = A$ are considered.

In terms of functional analysis, one has to solve the following equation if distributivity is considered:

$$S_1[x, T_4(y, z)] = T_5[S_2(x, y), S_3(x, z)]$$

which is equivalent to

$$S[x, \min(y, z)] = \min[S(x, y), S(x, z)] \ .$$

Classically extending the relation $(A \cap B) \cup (A \cap B^C) = A$ gives

$$S[T(x, y), T(x, ny)] = x \tag{1}$$

which is known (Alsina, 1985) to have no de Morgan solution.

A pexiderized extension of (1) gives

$$S[p(x, ny), i(x, y)] = x \ . \tag{2}$$

Solutions of (2) are explored for the case that S is a continuous t-norm associated to a strict negation n, p and i being symmetric functions on $[0, 1]^2$.

Bernard de Baets, Gent (Belgium)

Residuation in Fuzzy Set Theory

An extensively studied topic in fuzzy set theory is the solution of fuzzy relational equations. We put these equations in a broader perspective and study them from a purely lattice-theoretic point of view. We discuss the more general sup-$\mathcal{O}$ and inf-$\mathcal{O}$ equations, where $\mathcal{O}$ a logical operator. These equations can be considered as *polynomial lattice equations.* The main issues that have to be addressed are the following:

- the logical operators that can be considered,
- the types of complete lattices in which these equations can be solved,
- additional conditions that have to be imposed on these logical operators,
- a representation of the solution sets of these equations.

We deal with four primitive logical operators: conjunctors, disjunctors, implicators and co-implicators. The complete lattices in which these equations can be solved include the distributive, complete lattices of which all elements are either join-irreducible or join-decomposable, and/or meet-irreducible or meet-decomposable. Important representatives of the latter class of lattices are the product lattices $([0, 1]^n, \leq)$. The additional conditions imposed on the logical operators are mostly quite natural, and concern some morphism behaviour or some type of surjectivity of their partial mappings.

The representation of the solution sets of these equations is the key issue in solving fuzzy relational equations. If one succeeds in discovering an order-theoretic structure that can be used to fully describe the solution set of individual sup-$\mathcal{O}$ or inf-$\mathcal{O}$ equations, and that moreover behaves nicely under intersection and Cartesian product, then one is able to tackle more complex problems such as systems of equations, families of independent equations, ... We will show that *crowns* and *root systems* are the concepts one has been looking for. This representation is based on our knowledge of the extremal solutions. A compact description of these specific solutions requires a generalization of the theory of residuation from triangular norms to more general logical operators.

Radko Mesiar, Bratislava (Slovakia)

Universal Operations in Fuzzy Logic

The associativity as well as the commutativity of basic operations in fuzzy logic becomes a non-trivial problem when we have to deal with infinitely many inputs. The problem of an infinite extension of a conjunction operator (modelled by a t-norm) or of a disjunction operator (modelled by a t-conorm) may meet difficulties similar to those ones which occur by the extension of the common addition on the real line to the series sum. An axiomatic approach to a universal conjunction (disjunction) operator acting on arbitrary system of inputs from $[0, 1]$ is proposed. Any universal conjunction operator when acting on finite systems of inputs is shown to coincide with a t-norm, i.e. each universal conjunction operator is an extension of some t-norm. By duality, universal disjunction operators are extensions of t-conorms. Two kinds of possible extensions of a given t-norm T are proposed, namely the weakest T^* and the strongest T^{**}. Each T^* is shown to be a universal conjunction operator. However, $\min^* \neq \inf$. On the other hand, $\min^{**} = \inf$, but T^{**} needs not to be a universal conjunction operator. Conditions under which $T^* = T^{**}$ are given. Complete characterization of t-norms for which T^{**} is a universal conjunction operator is given, as well as a necessary condition, and a sufficient condition (namely, the right-continuity of T). The operator T^{**} with respect to a continuous T is described.

Jeffrey Paris, Manchester (UK)

Semantics for Fuzzy Logic

The assumption that an agent's belief value, $w(\theta)$ (or any of the other terms used to denote the agent's degree of confidence, or certainty) of a conjunction is a fixed function $F_\wedge$ of the belief values of the two conjuncts (and similarly for disjunctions and negations) is rather commonly applied in the construction of expert systems. Similarly one finds the same assumption being made in so called logics of vagueness where the values assigned to sentences are intended to represent their "degrees of truth". This raises the question of explaining what these values $w(\theta)$ actually mean, and why the functions F_θ employed to combine them are appropriate. Conventionally the choice of, say $F_\wedge$, has been guided by desirable properties (e.g. associativity, monotonicity) and such considerations lead us naturally to focus on the continuous t-norms for $F_\wedge$.

In my talk I consider possible semantics. or meanings, for $w(\theta)$ and the corresponding functions $F_\wedge$, etc. both for "belief values" and for "degrees of truth". In the former case $w(\theta)$ is interpreted as an approximation to the expected probability, $Prob(\theta)$ of θ and $F_\wedge$ etc. chosen to minimise the error resulting from using $w(\theta)$ in place of $Prob(\theta)$. In the second case, of $w(\theta)$ as "degree of truth" of θ, a meaning is given to $w(\theta)$ by identifying it with a notion of the *acceptability* of θ, more precisely the proportion of independent arguments which an agent could muster *for* θ, as opposed to *against* θ. Versions of "independent" are considered which gave *min* and *product* as the appropriate $F_\wedge$ etc.

Patrik Eklund, Umea (Sweden)
What is the Role of Logic in Biomedical Engineering?

Human reasoning within clinical decision making, or in general, within decision making and observations in the health care domain, involves not only intelligence, but also ethics and law that constrain decision making in various ways. Traditionally, statistics and probabilistic approaches, as means for supporting decision making processes, are well established in the health care domain, even in defining boundaries related to legality of actions. Compared to statistical methods, also numerical analytic means, as represented, e.g., by pattern recognition, clustering and also neural networks, provide tools for approximation and optimisation that are widely accepted by health care professionals. However, logical approaches, as building upon notions within resolution, algebra and equalities, do not seem to have experienced a breakthrough and acceptance so as to be recognised as providing supporting techniques in development of decision support systems.

Considering decision making as supported, respectively, by statistics, numerics and logic, we certainly agree that the conglomerates of methods provided in each domain overlap to some extent, and also that non-overlapping parts complement each other. However, very little seems to be known about similarities, formal relationships and interplay between various techniques. Some bridges, however, do exist. Propositional logic inference and neural network feed-forward can be seen as identical in a general framework, thus providing logical understandings of numerical computations, and similarly enabling parameter optimisations within expert systems. Conditionality in probabilistics is obviously related to implication, even if it is still not clear how formal transformation rules, rewriting probabilistic networks in to rule bases and vice versa, should be defined.

The difficulties within the many-valued logic communities to demonstrate applicabilitity, partly stems from hubris within the AI community some decades ago, when (heuristic) logic based expert systems, as strongly connected to knowledge acquisition techniques, were expected to have huge potentials for future decision support not only in health care but in general for almost any kind of decision scenarios. However, the logic community is also still to blaim in that the objectives in development of logical systems relate exclusively to soundness/completeness, satisfiability, NP-completeness, and notions alike. Sometimes it even seems in logic that we restrict to reaching understandings of what happens if we change "this" to "that" or "these" to "those" in the syntactic or semantic apparatus of some logical system, thus with very little relation to the objectives as understood and being enforced in human reasoning for a particular decision making situation.

Esko Turunen, Lappeenranta (Finland)
BL-algebras of Basic Fuzzy Logic

Basic logic algebras (BL-algebras) have been invented recently by Hájek [3] in order to provide an algebraic proof of the completeness theorem of a class of $[0, 1]$-valued logics familiar in fuzzy logic framework. BL-algebras arise as Lindenbaum algebras from certain logical axioms in a similar manner as MV-algebras (cf. [1, 2, 4]) do from the axioms of Łukasiewicz logic. In fact, MV-algebras are BL-algebras. The converse, however, is not true. It follows from a result of Höhle that BL-algebras with involutory complement are MV-algebras. In this study we start a similar study of BL-algebras as Belluce [1], Gluschankof [2], Hoo [4] and others have done in the theory of MV-algebras; there the basic tool is ideal theory while in BL-algebras, because of lack of a suitable algebraic addition, we have to deal with deductive systems. In MV-algebra theory, deductive systems and ideals are dual notions; there deductive systems are also called filters but, in order to avoid confusion, we prefer to talk about deductive systems. We introduce locally finite BL-algebras and prove that such algebras are MV-algebras. As one may expect, there is a one-to-one correspondence between deductive systems and congruence relations of a BL-algebra. We prove that a deductive system is maximal if, and only if, the corresponding quotient algebra is a locally finite MV-algebra. This fact implies one of the main result of our study: semi-simple MV-algebras are, in the sense of Chang and Belluce, the only BL-algebras that are representable by a system of fuzzy subsets of a set. However, as proved by Hájek [3], all BL-algebras are representable by linear BL-algebras. We have some preliminary results in order to characterize all linear BL-algebras. We introduce co-annihilators and prove some of their elementary properties; all these results will be an introduction for a future, more detailed analysis on BL-algebras. An extended article containing all proofs will be published in *Mathware and Soft Computer*.

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Costas A. Drossos, Patras (Greece)

Non-standard Methods in Many-valued Logics

In this talk, we examine the relationships of MV-algebras and non-standard mathematics (infinitesimal and Boolean analysis).

MV-algebras include, in an essential way, infinitesimals, e.g., $\mathrm{Rad}(A)$ consists of pure infinitesimals and the standard infinitesimal which is 0. Also Di Nola's representation theorem involves the nonstandard unit interval. After all these, it was felt that a systematic study of relationships of non-standard methods and MV-algebras was needed. We try to lay down some basic frameworks to study MV-algebras from a non-standard point of view. We introduce hyperfinite MV-algebras, and it is expected that these MV-algebras are the image through the Mundici Gamma functor, of a $*$-finite S-dense subset of ${}^{*}\underline{\gtrless}$. One can see that there are a lot of $*$-concepts. We have $*$-completeness, $*$-simple, $*$-Archimedian, $*$-semi-simplicity, $*$-good sequence, etc. It is shown that the $*$-Belluce representation theorem is not equal to the Di Nola representation theorem. It is shown that there is a hyperfinite MV-algebra which includes all rational points in $[0,1]$ but no irrational appears in it. This is very important in defining hyperfinite McNaughton functions. It is shown also that Chang's MV-algebra is external. A different proof is given that the Boolean power of $[0,1]$ is a semi-simple MV-algebra. Finally, we examine in which way stochastic MV-algebras can be defined.

Luisa Iturrioz, Lyon (France)

Non-functionally Complete n-Valued Systems Semantically Based on Posets

There are—at least—two traditional ways to study n-valued formal systems: functionally complete or not; namely, Post logics of order n or Łukasiewicz logics of order n (or even Łukasiewicz-Moisil logics of order n). In the best known n-valued systems, the basic algebra A given functionally completeness is an n-element chain $A = \{e_0, e_1, \ldots, e_{n-1}\}$.

In the past, many *computer applications* needed n-valued functionally completeness and this is one of the reasons why the theories of Post algebras and generalized Post algebras have been developed. Applications were closely related to the design of switching and electronic circuits, and in circuits usually the constant operations are at our disposal (at no extra cost). Also, let p be a prime number; in the Post algebra of order p the arithmetic operations of addition and multiplication can be defined. In this way, n-valued logics have been used in arithmetic units.

The contents of the book [1] is a typical example of our proposal. Even "the nomenclature for the so-called 'constants' $\{e_0, e_1, \ldots, e_{n-1}\}$ was chosen with n voltage levels in mind" [1, page 35].

In the n-valued case, constants are added to obtain functionally completeness. Another useful property given by the constants $\{e_0, e_1, \ldots, e_{n-1}\}$ is the following additional symmetry (George Epstein, 1960): A Post algebra of order n is dually isomorphic with itself under the mapping: $\beta(x) = \bigvee_{i=1}^{n-1}(d_i(x) \wedge e_i)$ where the $d_i(x)$ are generalized complementation operators.

At the present, *computer science applications* are more diversified than in the past, and more flexible (discrete) tools are needed. Following the non-functionally complete way, we intend to generalize the interesting and powerful works of Helena Rasiowa concerning Perception Logics: first, in Łukasiewicz style, that is, losing constants but keeping the symmetry; second, losing constants and symmetry. In both cases, only for *finite* posets. These first order systems are semantically based on posets which are interpreted as posets of co-operating intelligent agents.

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Teresa Alsinet Bernado, Lleida (Spain)
Fuzzy Unification

The unification process is usually associated with logic programming languages, which lets us compare the arguments of two predicates and infer new information through some deduction rule. In classical logic the unification problem is well-defined, and the set of techniques that implements it differs on the complexity associated with the process. A new problem arises when we add vague, incomplete or imprecise information to the system. It is necessary to define some measure in order to establish the degree of similarity between two fuzzy sets.

Our aim is to include the data type fuzzy set in a logic programming system based on a family of infinitely-valued logics. Within this context, we address the problem of unification involving fuzzy constants and linguistic variables in systems where a separation between general and specific knowledge can be made. We describe an algorithm for the construction of the most general fuzzy unifier of a pair of atomic formulae based on the semantics of terms. For this purpose, taking account of the distinction between general and specific knowledge, we define the semantic structure associated with variables and linguistic variables of a logic program and we specify the data structures used by the unification algorithm for its representation. With the aim of quantifying the inclusion degree between two fuzzy constants, we establish a similarity measure based on the interpretation of the membership function of a fuzzy set as the degree of similarity between the elements of the universal set and the set of prototypes. Moreover, we formalize the definitions of fuzzy substitution, fuzzy unifier and most general fuzzy unifier. Finally, taking the origin of the variable instances into account, we develop a measure of the unification degree of two atomic formulae through their most general fuzzy unifier.

With reference to future lines of work, we must consider the inclusion of functions in the fuzzy unification algorithm and the integration of the algorithm into the declarative programming environment for infinitely-valued logics. The first problem lies in defining the semantics of functions and the relationships between them and the fuzzy constants and the linguistic variables. To deal with the second problem, we must define the semantics of multiple-valued facts and rules of a logic program and develop a proof procedure. For this purpose, we must formalize the notion of interpretation, model, logical consequence and logical inference of a first-order multiple-valued language when the terms express vague, incomplete or imprecise information.

Finally, we would like to point out that the system may be particularly useful within the framework of fuzzy deductive data bases and diagnosis systems in which symptoms are not always described precisely. It may be also valuable for the implementation of dialog agents in multi-agent systems when vague, incomplete or imprecise information about the real world and approximately specified rules are needed.

Didier Dubois, Toulouse (France)
with Stefan Lehmke, Dortmund (Germany)
Fuzzy Logic = Many-valuedness + Partial Belief

Twenty years ago Zadeh [3] claimed that fuzzy logic

> ... is a logic of approximate reasoning [...] whose distinguishing features are (i) fuzzy truth-values expressed by linguistic terms, (ii) imprecise truth-tables.

This statement has been misregarded by some logicians who assume that fuzzy logic equals many-valued logics. Of course it is clear that the algebraic setting of fuzzy set theory is precisely the one of many-valued logics. However, it should not lead to a confusion between fuzziness in the wide sense and many-valuedness *stricto sensu*. When Zadeh points at fuzzy truth-values, he really refers to fuzzy sets of truth-values expressing imprecision, and not only to graded truth-values. Fuzzy truth-values are possibility distributions that combine many-valued truth and many-valued plausibility levels.

To support this claim, it can be shown that each logic possesses an embedded representation of belief, and not only a calculus of truth-values. In propositional logic, belief is two-valued (certainty versus lack of certainty) and is not self dual (to say that a proposition p is not believed is not equivalent to say that $\neg p$

is believed, contrary to probabilistic belief). Belief values are closely related to consequencehood and are not compositional [1], even in classical logic. We propose to consider fuzzy logic as relying on two ordered sets: a truth-set T that describes the range of propositional variables, and a plausibility set D that is the range of uncertainty levels. Fuzzy logic handles *labelled* formulas $[p, \lambda]$ where p is a formula in a given language and λ is a non-decreasing function from T to D, such that full truth is fully plausible. To write $[p, \lambda]$ is a means of declaring that p holds in some sense. Semantic evaluation involves both D and T. Namely, an interpretation of I is a model of $[p, \lambda]$ to a degree that represents the plausibility of the truth-value of p in I, as computed with λ. The degrees of modelhood are in the plausibility set and $\lambda(I(p))$ represents the degree of plausibility that I is the correct state of the world when the agent's knowledge is described by $[p, \lambda]$. It must be stressed that the set of fuzzy truth-values is not a truth-set. Labels are not compositional truth-values in a special many-valued logic. Fuzzy logic is both a many-valued logic *and* a logic of partial belief.

The proposed framework with labels attached to formulas encompasses the statement of propositions in classical logic (where $D = T\{0, 1\}$), of some signed formulas in many-valued logics (where truth is many-valued but plausibility is binary), and of weighted formulas in possibilistic logic (where truth is binary and plausibility is many-valued). It also offers a natural setting to extend possibilistic logic and many-valued logics conjointly [2] into a genuine fuzzy logic, that logicians may acknowledge as being a logic, and that remains faithful to the original motivation of Zadeh.

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Elena Tsiporkova, Plovdiv (Bulgaria)
with Bernard De Baets and Veselka Boeva, Gent (Belgium)
Possibility Theory in Modal Logic

In this talk, the modal logic interpretation of plausibility and belief measures defined on an arbitrary universe of discourse, as proposed by Harmanec, Klir, Resconi, St. Clair, and Wang, is further developed by applying notions from set-valued analysis. The attractiveness of this multi-valued interpretation lies in its natural analogy to the original approach of Dempster. In fact, as far as we manage to construct a multi-valued mapping that establishes a correspondence between the set of possible worlds and the set of atomic propositions, we automatically obtain without imposing any restrictions on the accessibility relation, that this mapping induces a plausibility measure and a belief measure. This multi-valued interpretation is explained in detail in the case of possibility and necessity measures. Moreover, it is shown that by restricting a model of modal logic in a non-trivial way, the possibility and necessity measures induced by this restricted model coincide with the conditional possibility and necessity measures of the measures induced by the original model, corresponding to the set used for restricting.

Stephan Lehmke, Dortmund (Germany)
with Didier Dubois and Henri Prade, Toulouse (France)

A Comparison of Particular Logics of Graded Incomplete Truth and Graded Incomplete Knowledge

We study the relationships between logics of *graded incomplete knowledge* (a prominent representative being *possibilistic logic*) and logics of *graded incomplete truth* (for which logical systems in the style of J. Pavelka are examples).

The two different types of logics are intended for the representation of two different aspects of *uncertainty* (in the linguistic sense) in natural language, which can be given by degrees.

In a logic which is intended purely for the representation of *graded incomplete knowledge*, it is assumed that a logical proposition is always either completely true or completely false, but for the knowledge to be represented, we are able to assign degrees of *uncertainty about* (respectively *trust in*) whether a statement from a knowledge base is indeed true.

In a logic which is intended purely for the representation of *graded incomplete truth*, we assume that not only can every proposition be true to a degree lying *between* complete truth and complete falsehood, but for the knowledge to be represented, we are able to assign degrees of truth which have to be assumed for the statements from a knowledge base to be considered valid.

In the present talk, we compare and differentiate the specific and unique properties of logics of graded incomplete truth and logics of graded incomplete knowledge, by demonstrating these for two specific examples: possibilistic logic with necessity-weighted formulae and Lee's fuzzy logic with truth values as weights.

Brunella Gerla, Salerno (Italy)
The Ulam Game and MV-entropy

In the Ulam game player A must guess an element of a finite set S, asking player B questions, who can only answer "yes" or "no" but can lie up to k times. The task is to find the secret element with as few questions as as possible. The Ulam game can be represented in Łukasiewicz logic and in particular it can be seen as a combination of classical information (questions are subsets of the set S) and multi-valued information (answers are elements of an appropriate MV-algebra).

This game is strongly connected with the problem of finding an error-correcting code, and in particular one can refer to a set on which a probability distribution is defined, in order to simulate better the random nature of the source. In this case it is possible to extend a result holding in the classical case. In fact, in the classical case, where the game is also called Twenty Questions Game, the best strategy is the balanced one, i.e., the strategy in which the questions "divide" the probability distribution; this can be shown with some observations on the entropy of the system.

In accordance with the definition recently developed in the literature, it is possible to introduce a generalization of the notions of probability and conditional probability, and so it is possible to give a definition of entropy that extends the classical one.

The result is that in the non classical case, too, best strategies (in the average case) are those that balance the probability.

Stefano Aguzzoli, Siena (Italy)
McNaughton Functions of One Variable for Automated Deduction in Łukasiewicz Logics

McNaughton functions play the same role in Łukasiewicz logics as boolean functions do in classical logic. Formulas in one variable are an important ingredient of automated deduction in many-valued logics. Here we present how it is possible to develop resolution calculi for the infinite-valued logic of Łukasiewicz based on suitable classes of formulas in one variable: these classes of formulas correspond to classes of McNaughton functions characterized by nice geometric properties. We introduce normal forms built upon these classes of formulas.

Manuel Ojeda Aciego, Malaga (Spain)
with Gabriel Aguilera Venegas and Inmaculada P. de Guzman, Malaga (Spain)

The TAS Reduction Method in MVL: A TAS Theorem Prover for Three-valued Logic

We present a new prover for propositional 3-valued logics, named TAS-M3, which is an extension of the TAS-D prover for classical propositional logic. As a *reduction-based* method, the power of TAS-M3 is based on processes which reduce the size of the formula. These processes filter the information contained in the syntactic structure of the formula to avoid as much distributions as possible. Roughly speaking, the idea is to get the information given by unitary satisfying assignments; this idea has proven to be extremely useful. TAS applies a sequence of transformations to the formula being considered. It is worth to note that the transformations are not just applied one after the other; through the efficient determination and manipulation of sets of unitary models of a formula, the method *investigates exhaustively* the formula, to detect if it is possible to decrease the size of the formula being analysed. Sets of unitary models are associated to each node in the syntactic tree of the formula, they can be considered the key tool of TAS methodology, since they are used to conclude whether the structure of the syntactic tree has or has not direct information about the validity of the formula. This way, either the method ends giving this information or, otherwise, it decreases the size of the problem before applying the next transformation. So, it is possible to decrease the number of distributions or, even, to avoid them all. The power of the method is based on the fact that every reduction process reduces the size of the formula and the branching part of the method follows a lazy strategy.

Neil Murray, Albany (USA)

Parameterized Prime Implicant/Implicate Computations for Regular Logics

Prime implicant/implicate generating algorithms for multiple-valued logics are introduced. Techniques from classical logic not requiring large normal forms or truth tables are adapted to certain “regular” multiple-valued logics. This is accomplished by means of signed formulas, a meta-logic for multiple valued logics; the formulas are normalized in a way analogous to negation normal form. The logic of signed formulas is classical in nature.

The presented method is based on path dissolution, a strongly complete inference rule. The generalization of dissolution that accommodates signed formulas is described. The method is first characterized as a procedure iterated over the truth value domain $\Delta = \{0, 1, \dots, n-1\}$ of the multiple-valued logic. The computational requirements are then reduced via parameterization with respect to the elements and the cardinality of Δ.