Report on the Dagstuhl-Seminar 9510

Cellular Automata

Organizers:
Jozef Gruska (Bratislava)
Hiroshi Umeo (Osaka)
Roland Vollmar (Karlsruhe)

March, 6 – 10, 1995

Preface

Since the basic work of J. von Neumann who introduced cellular automata to study the notion of self reproduction cellular automata have been investigated as theoretical models for

- the study of (one sort of) parallel processing systems
- the physical world
- for the study of behaviour of complex systems
- for the study of "artificial life".

During the last decade an increasing interest on this topic can be observed starting with the investigations of Wolfram.

The talks and the discussions during the seminar contributed to the state of the art mainly concerning the following aspects:

- Cellular automata as information processing systems
- Cellular automata as dynamical systems
- The phenomenological and algorithmic complexity of cellular automata
- Generalizations and modifications of cellular automata
- Cellular automata as models in Physics

During the meeting also talks about the further work of the IFIP Working Group 14.5 on Cellular Automata took place. It was the common conviction that similar seminars and workshops should be organized in a somewhat regular manner.

The stimulating atmosphere combined with a perfect organization by the staff of Schloß Dagstuhl was appreciated very much by the participants.

The seminar report at hand has been put together by Th. Worsch.

R. Vollmar

Participants

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Program

Monday, March 6:

Klaus Sutner: De Bruijn Automata and the edge of chaos
Shinji Takesue: Thermodynamic Behaviour of One-Dimensional Reversible Cellular Automata
Erich Prisner: Parallel Chip Firing on Digraphs
Ivan Korec: Directions of reconstructability of one-dimensional cellular automata
Fritz von Haeseler: Selfsimilarity structure and automaticity of orbits of cellular automata
Guentcho Skordev: Self-similarity and automaticity of orbits of a class of cellular automata
Satoshi Takahashi: Multifractal Formalism for Sofic Measures

Tuesday, March 7:

Max Garzon: Observability in Cellular Automata and Neural Nets
Veronique Terrier: Real Time One Way Cellular Automata
Martin Kutrib: Real-Time One-Way Pushdown Cellular Automata
Burton Voorhees: Surjectivity of Cellular Automata Rules
Bruno Durand: Reversibility and dimension-sensitive properties of cellular automata
Howard Gutowitz: Criticality and complexity in cellular automata
Jean-Baptiste Yunès: Fault tolerant solutions to the firing squad synchronization problem
Frédéric Geurts: Compositional Approach of Cellular Automata

Wednesday, March 8:

Eric Goles: Sand Piles and chip firing games
Max Garzon: Real-valued Computation Using Local Computation
Thomas Worsch: On classes of transition functions satisfying a Frobenius law
Jacqueline Signorini: A programming environment for cellular computers
Umberto Pesavento: Von Neumann's Universal Constructor

Thursday, March 9:

Jacques Mazoyer: Computations on 1-dimensional Cellular Automata
Iwo Bialynicki-Birula: Relativistic wave equations as unitary cellular automata
André Barbé: Coarse-growing invariant orbits of one-dimensional Z_p-linear cellular automata
Maurice Margenstern: The halting problem for Turing machines: decidability versus undecidability, a survey
Bruno Martin: An intrinsic universal cellular automaton
Burton Voorhees: Three remarks on Additive Cellular Automata
Patrizia Mentrasti: Cryptography with Cellular Automata

Ginaluca Tempesti: Self-reproduction in cellular automata: software and hardware realizations
Wild cat session: contributions by B. Durand, M. Garzon, I. Korec, J. Mazoyer, L. Priese, K. Sutner, R. Vollmar, Th. Worsch

Friday, March 10:

Giancarlo Mauri: Cellular automata in the fuzzy background
Gianpiero Cattaneo: Some tools to classify one dimensional Cellular Automata on the basis of their dynamical behavior
Jacques Mazoyer: Synchronization
Ivan Korec: Elementary theories of generalized Pascal triangles

Coarse-graining invariant orbits of one-dimensional Z_p-linear cellular automata.

André M. Barbé
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(joint work with F.v. Haeseler, H.O. Peitgen, G. Skordev - Univ. Bremen)

We first introduced the notion of coarse-graining of the orbit of a one-dimensional CA over Z_2: the two-dimensional binary state-time orbit is partitioned in square tiles of 2×2 cells, and each tile is replaced by one new cell whose state-value is the sum mod 2 of the states of the four cells covered by that tile. By this coarse-graining operation, a new orbit is obtained which satisfies the original local CA-rule. Question: are there orbits which are invariant under the two possible different 2×2 square tiles? Answer: yes. Their number is finite. Examples for two different CA-rules were presented. The initial configurations that generate coarse-graining invariant (CGI) orbits are 2-automatic, and have correlation-functions and Fourier-transforms resembling those of the paperfolding sequence. The two-dimensional CGI-orbit patterns themselves are of a particular nature: apart from the trivial null-solution and some Sierpinsky-gasket like structure, they are all quasiperiodic. These solutions have an underlying structure which becomes visible by 'EXOR-ing' the orbit pattern with a shifted version of itself.

Then we presented a 4-point generalization, by considering: (1) CA over Z_p with $p^n \times p^n$ tiling-size (p prime); (2) arbitrary tile-shapes; (3) arbitrary Z_p-weighting of all cells in a tiling; (4) orbit invariance modulo a shift. This leads to a CGI-problem with many control parameters. Possible solutions are of the following type: periodic, quasiperiodic, self-similar, quasirandom, randomlike.

References:

1. A. Barbé, F.v. Haeseler, H.O. Peitgen, G. Skordev: 'Coarse-graining invariant patterns of one-dimensional two-state linear cellular automata', Report nr. 319, Institut für dynamische Systeme, Universität Bremen, September 1994. (to appear in Int. J. Bifurcation & Chaos, Vol. 5, 1995).

2. A. Barbé: 'Coarse-graining invariant orbits of one-dimensional Z_p-linear cellular automata', ESAT/SISTA report 94-72, Dept. of Electrical Engineering, K.U. Leuven, March 1995.

Relativistic wave equations as unitary cellular automata

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Relativistic wave equations (Weyl, Dirac, and Maxwell) are implemented as cellular automata on a three dimensional lattice. In order to satisfy the requirement of probability conservation, the automata are assumed to be unitary. That means that during each update the sum of the squared moduli of the wave functions summed over the whole lattice is preserved. In addition, the update function is assumed to satisfy the requirement of locality; the new value at a given lattice site depends only on the value at the neighboring sites. It turns out that these requirements are difficult to satisfy. For some lattice structures (for example, for a simple (6 neighbors) cubic lattice) no solutions exist. An explicit solution is given for the body-centered (8 nearest neighbors) cubic lattice and it is shown that in the limit of the lattice constant shrinking to zero, the unitary automaton reproduces the continuous relativistic wave equation. The content of the lecture is based on the paper: Weyl, Dirac, and Maxwell equations on a lattice as unitary cellular automata, Physical Review D49, 6920 (1994).

Some tools to classify one dimensional Cellular Automata on the basis of their dynamical behavior

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The study of one dimensional Cellular Automata as dynamical system carried out by Wolfram, induced him to introduce an empirical classification of these models [4]. Since then, especially when Wolfram classification was shown to be undecidable [3], many efforts have been spent on trying to give a formal classification based on the dynamical behavior of the automata and rigorous and effective criteria to establish the belongings of a given CA to a determinate class, without evolving it. We introduce a formal classification of quiescent one dimensional CAs, based on the behavior of finite configurations on a null background. This classification has been shown to be effective when restricted to quiescent elementary CAs; furthermore we give effective criteria to determine the qualitative dynamics of CAs belonging to some of the classes introduced.

Moreover we give some useful tools to classify elementary CAs and characterize their chaotic dynamics.

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A subclass of elementary Cellular Automata, equivalent to a particular class of Neural Networks (NN) is studied. The NNs are dependent on four parameters; it is observed which subspace of the Rule Space is obtained varying the parameters and what are the characteristics of the rules in this subset. In addition, a few of nice and "magic" regularities in this subset is noted and explained.

Finally we give an algebraic characterization of elementary CAs and study the relations between algebraic structure and dynamical behavior of a CA.

Bibliography

[1] G.Braga, G.Cattaneo, P.Flocchini, G.Mauri, *Complex Chaotic Behavior of a class of Subshift Cellular Automata*, Complex Systems, **7** (1993) 269-296.

[2] G. Braga, G. Cattaneo, P. Flocchini, C. Quaranta Vogliotti, *Pattern Growth in Elementary Cellular Automata*, to appear in Theoretical Computer Science A, vol. 145 July 1995.

[3] K. Culik, S. Yu, *Undecidability of CA Classification Schemes* Complex Systems **2** (1988) 177-190.

[4] S. Wolfram, *Universality and Complexity in Cellular Automata*, Physica D **10** (1984) 91-125.

Reversibility and dimension-sensitive properties of cellular automata

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We consider cellular automata (CA) as functions that transform a configuration into another one. A configuration associates a state to each cell of Z^n. The set of states is finite. When CA are used as computational devices they are often restricted to the set of periodic configurations $\mathcal{P}$, or to the set of finite configurations $\mathcal{F}$.

We compare the class of injective (resp. surjective, bijective) CA defined on all configurations (resp. on $\mathcal{F}$, on $\mathcal{P}$). Some inclusions are proved using topology, others use combinatorial methods. Some of them are open for 2D CA. $G|_{\mathcal{P}}$ bijective $\Rightarrow$ G bijective is true in 1D and false in 2D otherwise G bijective would be a decidable property which is false (Kari 89). This non-inclusion property has never been proved directly without the use of the recursion theory. Other undecidability results are known in 2D: the surjectivity problem (Kari 89, Durand 93), the injectivity problem on periodic configurations (Durand 93). Some others are open such as the surjectivity problem on periodic (finite) configurations or the injectivity problem on finite configurations.

Observability in Cellular Automata and Neural Nets

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It is shown that arbitrary locally finite discrete neural networks of bounded radius over a state set which is an abelian group are observable (have the shadowing property) in the sense that pseudo-orbits obtained by small perturbations of an orbit are approximated by actual orbits. The class includes discretizations of some analog networks, a large class cellular automata, and a large set of linear maps on a one-dimensional grid. It follows that the true qualitative behavior of these dynamical systems can be observed to infinite precision on computer simulations, despite unavoidable discretization and approximation errors.

Real-valued Computation Using Local Computation

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We examine the problem of evaluating real-valued functions in one or unboundedly many iterations with informationally feasible computational models such as those afforded by cellular automata and discrete neural networks. They allow exact computation of certain functions (unlike Turing computatbility) and of discontinuous functions in variable finite time. Now, feedforward networks are well known to approximate any continuous function through any fixed number of iterations to an arbitrary degree of accuracy. We examine the approximating power of neural networks through an unbounded number of iterations. We prove that every continuous dynamical system can be approximated through all iterations, by both finite analog and boolean networks, when one requires approximation of arbitrary exact *orbits* of the given map.

However, this result no longer holds when the orbital behavior of approximant neural networks is not observable exactly due to the presence of random noise in analog activations or digital implementations. Neural nets can nonetheless approximate large families of both continuous (including chaotic maps) and discontinuous maps (including baker maps and maps with dense periodic points). A precise characterization is an open problem.

Compositional Approach of Cellular Automata

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We propose to study complex dynamical systems by a compositional approach. Given a system, we decompose it to obtain individual subsystems; for each of them, we study invariance and attraction properties; finally, we deduce the global properties by composition of the individual analyzes.

Composition operators are defined: sequential composition ( ), free product ($\times$, without interaction), sum ($\oplus$, nondeterministic choice), union ($\cup$, set-theoretic union of all possibilities), and connected product ($\otimes$, explicit interaction between components).

Since CA are massively parallel systems with interaction between cells, the connected product seems a natural operator. Our concern is the study of local composition composed with this connected product: f and g being two local transition functions, $\star$ (resp. $\diamond$) being a local (resp. global) operator, we search for an algebraic expression of the form $\otimes(f \star g) = \otimes(f) \diamond \otimes(g)$ such that the dynamics of the $\diamond$-composition is analytically characterized.

Up to now, we have a chacterization of invariants and attractors of -, $\times$-, $\oplus$-, and $\cup$-compositions under some assumptions. We are working on the extension of these results to $\otimes$-compositions.

Conjectures have been proposed linking $\vee$-local and $\otimes$-global operations. In particular, the $\vee$-composition of symmetric rules (e.g. left and right shifts) generates complex orbits. Since the sum shows the same kind of behavior in classical dynamical systems, we weaken the local disjunction to obtain a local sum. With the help of different tools (entropy, boolean derivative, probabilistic CA), we show that this system has complex dynamics.

In conclusion, our compositional approach shows that very simple systems composed together can generate very rich dynamics. We propose theoretical results but the approach has to be developed to include more systems. Indeed, some specific cases can be analyzed in a straightforward way, but the inherent distributed interaction of CA entails some difficulties that probabilities or algrebra could solve.

Sand Piles and chip firing games

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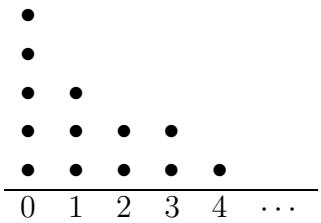
Given a one-dimensional lattice N we define a sand-pile as a non-increasing configuration of integers; for instance:



$= 5\,3\,2\,2\,1$

So, a sand pile is a word $w_0 w_1 \cdots w_i$ such that $\sum_{i\geq 0} w_i = n$, $w_i \geq w_{i+1}$, $w_i \in \mathsf{N}$. We define a local rule at site i:

$$\tau_i(w) = w_0 \cdots w_{i-1}, w_i - 1, w_{i+1} + 1, w_{i+2} \cdots \text{ iff } w_i - w_{i+1} \geq 2$$

We prove that for all w the local dynamic converges to a fixed point (w^* which can be characterized). Further, if the number of grains of

$$n = \frac{k(k+1)}{2} + k' \text{ where } k' \leq k$$

the number of steps from the sand pile $w^0 = (n, 0, 0, \dots)$ to the fixed point is

$$\binom{k+3}{3} + kk' - \binom{k'}{2}$$

On the other hand, previous model is equivalent to the following game: given a configuration $x = \cdots x_i \cdots$ a legal move is $x'_i = x_i - 2, x'_{i\pm 1} = x_{i\pm 1} + 1$ when $x_i \geq 2$.

In a general undirected graph $G = (V, E)$ the local rule is $x'_i = x_i - d_i$ and $x'_j = x_j + 1$ for all $j \in V_i$ (V_i neighborhood of vertex i). When this rule is applied in parallel we prove that for trees any initial configuration converges to fixed points or two periodic configurations. For general graphs may appear non-polynomial cycles.

Criticality and complexity in cellular automata

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Is there an Edge of Chaos, and if so, can evolution take us to it? Many issues have to be settled before any definitive answer can be given. For quantitative work, we need a good measure of complexity. We suggest that *convergence time* is an appropriate and useful measure. In the case of cellular automata, one of the advantages of the convergence-time measure is that it can be analytically approximated using a generalized mean field theory.

In this paper we demonstrate that the mean field theory for cellular automata can 1) reduce the variablity of behavior inherent in the λ-parameter approach, 2) approximate convergence time, and 3) drive an evolutionary process toward increasing complexity.

Selfsimilarity structure and automaticity of orbits of cellular automata

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Let $V = \{v_0, \dots, v_N\}, n \geq 1$, be a finite set of symbols, where v_0 is a distinguished element. Let $\Sigma(V) = \{\underline{a} : \mathsf{Z} \to V\}$ be equipped with the product topology induced by the discrete topology on V. A cellular automaton is a continuous map $A : \Sigma(V) \to \Sigma(V)$ which commutes with the shift map $\sigma : \Sigma(V) :\to \Sigma(v), \sigma(\underline{a})(i) = \underline{a}(i+1)$.
A theorem of Hedlund (1969) states: Let A be a cellular automaton, then there exist natural numbers d_1, d_2 and a map $\phi : V^{d_1+d_2+1} \to V$ such that

$$A(\underline{a})(i) = \phi(\underline{a}(i-d_1), \dots, \underline{a}(i), \dots, \underline{a}(i+d_2))$$

holds for all $\underline{a}$ and $i \in \mathsf{Z}$. The mapping ϕ is called generating function. On the other hand, any $\phi : V^d \to V$, where d is a natural number, defines via $A(\underline{a})(i) = \phi(\underline{a}(i-d+1), \dots, \underline{a}(i))$ a cellular automaton A.

In the following, we consider cellular automata with generating function $\phi : V^d \to V$ such that $\phi(v_0, \dots, v_0) = v_0$.

Let $(\mathcal{H}(\mathsf{R}^2), h)$ denote the metric space of all non-empty compact subsets of the euclidian plane, where h denotes the Hausdorff distance induced by the euclidian metric. Furthermore, let $I = [0,1]^2$ denote the unit square and $I(s,t) = \{(x+s, y+t) \,|\, (x,y) \in I\}$.
The set

$$X(A, \underline{a}, n) = \bigcup_{\substack{s \in \mathsf{Z} \\ 0 \leq t \leq n-1 \\ A^t(\underline{a})(s) \neq v_0}} I(s,t)$$

is called the n-th orbit representation of $\underline{a}$ w.r.t. the cellular automaton A. is called the n-th orbit representation of $\underline{a}$ w.r.t. the cellular automaton A.
Let $p \geq 2$ be a natural number and define the map $\pi_p : \Sigma(v) \to \Sigma(V)$ by

$$\pi_p(\underline{a})(i) = \begin{cases} \underline{a}(j) & \text{if } i = pj \\ v_0 & \text{otherwise.} \end{cases}$$

A cellular automaton is called p-Fermat in $\underline{a}$ if $A^{p^n}(\pi_p(\underline{a})) = \pi_p(A^n(\underline{a}))$ holds for all $n \in \mathsf{N}$.
Theorem. Let A be p-Fermat in $\underline{a}$. Then

$$\lim_{n \to \infty} \frac{1}{p^n} X(A, \underline{a}, P^n) = X_\infty(A, \underline{a})$$

and the limit object, $X_\infty(A, \underline{a})$, is described as a geometrical substitution.

Associated with a cellular automaton is a formal Laurent series defined by

$$G(A,\underline{a}) = (X,Y) = \sum_{\substack{s \in \mathsf{Z} \\ t \in \mathsf{N}_0}} A^t(\underline{a})(s) X^s Y^t.$$

Theorem. Let A be p-Fermat in $\underline{v} = (\dots, v_0, v_0, v, v_0, v_0, \dots)$, where $v \in V$. Then $G(A,\underline{v})(X,Y)$ is p-automatic.

Directions of reconstructibility of one-dimensional cellular automata

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Reconstructibility will be a generalization of reversibility in one-dimensional cellular automata (1dCA) and generalized Pascal triangles. (The later are defined in the abstract "Elementary theory of generalized Pascal triangles", and correspond to computations of 1dCA from finite initial configurations.)

Computations of 1dCA can be imagined in the discrete $(1+1)$-dimensional space-time; there will be one coordinate axis for space and another one for the time. This is an analogy of $(3+1)$-dimensional space-time in physics.

We shall deal with the oriented (euclidean) plane. The directions in the plane will be determined either by vectors or by the angles (in degrees) between the direction "upwards" and the considered direction; so $0°$ or $360°$ will be the North (upward) direction, $90°$ the East direction, $180°$ the South direction and $270°$ the West direction. A direction α will be called *rational* if $\tan(\alpha)$ is rational or undefined.

Directions of reconstructibility will be defined for arbitrary set $\mathcal{F}$ of partial functions on the set Z^2 of the lattice points of the plane. However, we shall consider them mainly in the case when $\mathcal{F}$ is the set of computations of a 1dCA. In the definition the following notation will be used:

$$\mathrm{HP}(A,b) = \big\{(x_1,x_2) \in \mathsf{Z}^2 \;\big|\; a_1x_1 + a_2x_2 + b < 0\big\}$$

is the set of lattice points of the open half-plane associated to the nonzero vector $A = (a_1,a_2)$ and a real b. (If A is considered as the direction of time then $\mathrm{HP}(A,0)$ represents "the past".)

Definition 1 *Let $\mathcal{F}$ be a set functions whose domains are subsets of Z^2, let A be a nonzero vector and let $\mathbf{X} \subseteq \mathsf{Z}^2$.*

(i) *We shall say that $\mathcal{F}$ is $\mathbf{X}$-reconstructible if for all $f,g \in \mathcal{F}$ and $U,V \in \mathsf{Z}^2$ the following implication holds:*

$$(\forall Y \in \mathbf{X})\big(U+Y \in \mathrm{Dom}(f) \wedge V+Y \in \mathrm{Dom}(g) \wedge f(U+Y) = g(V+Y)\big) \implies f(U) = g(V).$$

(ii) *We shall say that $\mathcal{F}$ is /finitely/ reconstructible in the direction A if there is a positive real ε and a /finite/ subset $\mathbf{X}$ of HP(A,ε) such that $\mathcal{F}$ is $\mathbf{X}$-reconstructible.*

Roughly speaking, computations of one-dimensional CA are usually displayed so that they are finitely reconstructible in the direction 180°. Then reversibility is finite reconstructibility in the direction 0°.

As an application, a new construction of reversible 1dCA is given: To every two opposite rational directions of finite reconstructibility α, $\alpha + 180°$ of (the set of computations of) a 1dCA (not necessarily reversible one) a reversible 1dCA can be constructed. The construction can be applied also to Pascal's triangle modulo n and the directions 90°, 270°.

Elementary theories of generalized Pascal triangles

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To every finite algebra $\mathcal{A} = \langle \mathbf{A}; *, \mathrm{o}\rangle$ (with $\mathbf{A} \subseteq \mathsf{N}$ to avoid some technical problems) such that $\mathrm{o} * \mathrm{o} = \mathrm{o}$ and every word $w \in \mathbf{A}^+$ we shall construct $G = \mathrm{GPT}(\mathcal{A}, w)$ as follows. The initial word w is written down into the initial row (with spaces between its letters). Every further row is formed from the previous one by the operation $*$ analogously as $+$ (and at the margins o analogously as 0) is used in the classical Pascal triangle. More formally, if $w = w_0 \dots w_{|w|-1}$ then the function $G = \mathrm{GPT}(\mathbf{A}, w)$ will be defined by the following formula:

$$G(x,y) = \begin{cases} \text{undefined} & \text{if} \quad x + y < |w| - 1, \\ w_x & \text{if} \quad x + y = |w| - 1, \\ \mathrm{o} * G(0, y-1) & \text{if} \quad x = 0,\ y \geq |w|, \\ G(x-1, 0) * \mathrm{o} & \text{if} \quad y = 0,\ x \geq |w|, \\ G(x-1, y) * G(x, y-1) & \text{if} \quad x + y \geq |w|,\ x > 0,\ y > 0. \end{cases}$$

The system of coordinates is chosen so that the whole GPT lies in the first (i.e., "positive") quadrant of the plane. For example, if $\mathbf{Z}_n$ is the additive group modulo n then $B_n = \mathrm{GPT}(\mathbf{Z}_n, 1)$ is the Pascal triangle modulo n; we have $B_n(x,y) = \binom{x+y}{x} \mathrm{MOD}\, n$ for all $x, y \in \mathsf{N}$.

The simplest mathematical structure associated to a GPT G is the partial groupoid $\langle \mathsf{N}; G\rangle$. We can also consider the relational structure $\langle \mathsf{N}; \mathrm{EqG}\rangle$, where EqG corresponds to the equivalence relation induced by G, i.e.

$$\mathrm{EqG} = \big\{(x,y,z,w) \in \mathrm{Dom}(G) \times \mathrm{Dom}(G) \;\big|\; G(x,y) = G(z,w)\big\}.$$

Another possibility is to consider many-sorted structures $\langle \mathsf{N}^{\mathrm{x}}, \mathsf{N}^{\mathrm{y}}, \{0,1,2\}; G\rangle$ or $\langle \mathsf{N}^{\mathrm{x}}, \mathsf{N}^{\mathrm{y}}; \mathrm{EqG}\rangle$ where $\mathsf{N}^{\mathrm{x}} = \mathsf{N}^{\mathrm{y}} = \mathsf{N}$, G is considered as a mapping of $\mathsf{N}^{\mathrm{x}} \times \mathsf{N}^{\mathrm{y}}$ into $\mathbf{A}$ and

EqG is considered as a subset of $\mathsf{N}^{\mathrm{x}} \times \mathsf{N}^{\mathrm{y}} \times \mathsf{N}^{\mathrm{x}} \times \mathsf{N}^{\mathrm{y}}$. In every case above we can add some operations or relations, e.g. the successor, $\leq$ or $+$. Results about elementary theories of such structures are presented. For example:

Theorem 1 *Let $\mathcal{B} = (\{2, 0, 1\}; *, 2)$, where $2*0 = 0$, $0*0 = 1$, $1*0 = 2$ and $x*y = x$ otherwise (i.e., if $y \neq 0$), and let $G = \mathrm{GPT}(\mathcal{B}, 0)$. Then:*

(i) *The operations $+$, $\times$ on N are first order definable in the structure $\langle \mathsf{N}; G \rangle$.*

(ii) *The elementary theory of $\langle \mathsf{N}; G \rangle$ is undecidable.*

(iii) *The elementary theory of $\langle \mathsf{N}^{\mathrm{x}}, \mathsf{N}^{\mathrm{y}}, \{0, 1, 2\}; G \rangle$ is decidable.*

The same hold if we replace G by EqG.

Theorem 2 *Let $n > 1$ and B_n be the Pascal triangle modulo n. Then:*

(i) *The elementary theory of $\langle \mathsf{N}; B_n, + \rangle$ is decidable if and only if n is a prime power.*

(ii) *The operations $+$, $\times$ are definable in $\langle \mathsf{N}; B_n \rangle$ if and only if n is divisible by two distinct primes.*

Real-Time One-Way Pushdown Cellular Automata

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Since the historical precedent for a fixed amount of memory per cell (unbounded) cellular automata (CA) have to be defined over an infinite space to obtain computational universality. Therefore, the number of required processors depends on the length of input data and, additionally, may increase during the computation.

From a more practical point of view an infinite number of processors seems to be fairly unrealistic. On the other hand Turing acceptors are computationally universal devices which have one processor only and additionally an infinite storage tape. For this reason and due to the possible speed-up gained in parallelism we introduce the pushdown cellular automata (PDCA), in which each cell is now a deterministic pushdown automaton.

Here we restrict ourself to one-way information flow (OPDCAs) and, especially, to real-time computations.

OPDCAs and related notations are formally defined. Some results concerning the language recognition capabilities of one-way pushdown cellular automata are presented. Real-time acceptors for several languages are given and the relationship to other types of acceptors is studied. In particular, we compare real-time OPDCAs to finite state machines, linear bounded automata, Turing machines and classical one-way cellular automata without pushdown memory.

The halting problem for Turing machines: decidability versus undecidability, a survey

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We remind Turing's original proof of the undecidability of the halting problem which does not involve a universal machine. We draw a sketchy picture of results about decidability versus undecidability in various settings: diophantine equations, word problem, PCP, tag-systems, tilings, cellular automata and planar Turing machines.

We focuse our attention on "classical" Turing machines, i.e. deterministic ones with a single bi-infinite tape and a single head. We mention Rogozhin's results on universality of very small (in program size) Turing machines.

We define a general notion of *decidability criterion* on a given set of Turing machines with a *frontier value*: under the frontier value, the halting problem is decidable and starting from this value, there are always universal Turing machines with greater or equal criterion value.

We detail the technique used for establishing two frontier results on Turing machines on $\{0, 1\}$, first for all machines on this alphabet and afterwards on non-erasing machines. The results are:

Defining the colour of instruction ixMyj (current state, input letter, move to perform, output letter, next state) as triple xMy, and the colour number of a machine as the number of distinct instruction colours occuring in its program, the colour number is a decidability criterion with 3 as a frontier value for all machines on $\{0, 1\}$ (Pavlotskaya) and with 5 as a frontier value for non-erasing machines on $\{0, 1\}$ (Margenstern).

Defining the laterality number as the least number of instructions with the same move, this number provides also a decidability criterion with 2 as a frontier value for all Turing machines on $\{0, 1\}$ (Pavlotskaya and Margenstern) and with 3 as a frontier value for all non-erasing ones (Margenstern).

For the laterality number, it is also a criterion on richer alphabet with 1 as a frontier value for all machines on an alphabet having at least 3 letters (Margenstern-Pavlotskaya) and with 2 as a frontier value for non-erasing machines on an alphabet containing 3 letters (Margenstern), whatever way we should extend the notion of non-erasing on an alphabet with more than 2 letters (in fact by an *order*, not necessarily total, on the letters of the alphabet with blank symbol as a minimum).

An intrinsic universal cellular automaton

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Cellular automata are a model of parallel computation capable of universal computation since their introduction by J. von Neumann. Their computational power is well known in two dimensions and A.R. Smith proved first that one dimensional cellular automata were capable of simulating any Turing machine.

This is sufficient to prove that, with the formalism introduced by H. Rogers, cellular automata form a universal programming system. But it was an open problem whether there exists a unidimensional cellular automaton capable of simulating the behavior of any given cellular automaton on any given -finite- initial configuration as some universal Turing machine do. This problem was first solved in 1987 by J. Albert and K. Culik. They designed a universal unidimensional cellular automaton which is slower than $\mathcal{A}$ the cellular automaton to be simulated of a factor quadratic in the number of the states of $\mathcal{A}$. They also require $\mathcal{A}$ to be a one-way and totalistic cellular automaton.

We propose here an enumeration -or Gödel numbering- of totalistic cellular automata and a universal cellular automaton. This one can simulate any given two-ways totalistic cellular automaton on any -finite- initial configuration. It works in quasi-linear time instead of quadratic time.

This, together with a composition function and the definition of a computation allows us to prove that cellular automata form an acceptable programming system. As a consequence, we get all the classical results in computability theory. We also observe that it is possible to define Kolmogorov complexity by means of cellular automata. This observation is not surprising as Kolmogorov complexity is independent with respect to the numbering and thanks to the Roger's isomorphism between cellular automata and Turing machines.

Cellular automata in fuzzy backgrounds

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The main purpose of this work is to understand some limitations introduced by the classical definitions of CA. To this end, we have defined a new model of CA (Fuzzy CA) which allows the observation of interesting "chaotic" properties of elementary CA. To date neither a formal nor a precise definition of "chaos" in Cellular Automata (CA) exists; we believe that the proposed model provides a "sharper" tool to detect which properties can be associated to a "chaotic" behavior. We also define a measure (Rule Entropy) which gives information about the CA's dynamics solely on the basis of the rule table and provides theoretical explanations to some of the empirical observations.

Computations on 1-dimensional Cellular Automata

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Designing cellular automata in order to achieve a dedicated task is an old problem. This engineer point of view may concern specific parallel questions (French flag, Firing squad, ...) or, more generally, computational tasks. In this last case, one-dimensional automata (1DCA for short) appear as a 'synchronous' model of massively parallel computation.

Many difficulties arise in order to define inputs and outputs. The input question may be efficiently solved in deciding to put the data on the first diagonal of the space-time diagram. The output question is more involved. A way to escape the difficulty is to limit oneself to language recognition. It has been intensively done, and an astonishing and probably very difficult open question remains: on a device such that the number of working cells is the input length (bounded computations), can all possible computations be achieved in real time? Coming back to 1DCA as functions computing, it was proved that computations always can occur on a *trellis* , which does not need to be regular.

We will call *grid* any trellis (regular or not). Local computations take place on the nodes of such a virtual device. This allows to move calculi on the space-time diagram defining the underlying grid. Thus, composing functions becomes putting together two grids in such a way that the border of the first one (on which the output of the first part of the calculus is to be found) is the border of the second one (on which lays the input of the second part of the calculus, namely the previous output). Moreover, looking to the methods involved in the Firing Squad, one may construct infinite families of grids, thus, infinite compositions of functions.

It is proved that any recursive function can be computed using grids. But a complexity analysis shows that using families of grids wastes time. We present a method which takes in account delays in transmission or computation. To avoid accumulation of data, small areas are frozen, what implies the possibility to dynamically set up grids as soon as possible. To end we observe that, in this frame, we never need any global synchronization process.

Synchronization

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The firing squad synchronization problem is to construct a cellular automaton such that any finite configuration with only one active distinguished cell evolves to synchronization.

In the one dimensional case, the first solutions are due to M. Minsky. A question of optimality appears: what is the number of states needed to obtain a minimal time

solution. Results are: 8 states (R. Balzer), then 6 states (J. Mazoyer). We know that there does not exist any 4 states solution.

We present generalizations to the graphs, and the problematics of what may be a minimal solution (K. Kobayashi). We also indicate some results of T. Jiang on synchronization of non uniform graphs, and some of our own results on this topic.

Cryptography with Cellular Automata

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We investigate the use of cellular automaton frameworks in designing data encryption systems. In particular, we focus on the requirements yielding from a standard cryptography analysis and we thus state the necessity of invariant discovering results for the class of finite cellular automata. We also recall some relevant results and open problems dealing with the connections between the theory of finite invertible cellular automata and cryptography. For instance, we recall the co-RNP-completeness of the invertibility problem. As central result, we design a new crypto-system with secret key secret key which uses classes of invertible finite cellular automata and we then prove that it is possible to construct these classes in order to generate a key space having sufficiently large size. We discuss the system security under different attacks. We observe that our approach can be extended to any other invertible automata generation rule and also that it is independent of the support-space dimension. Consequently, the system works correctly and efficiently on both 1-dimensional and multidimensional support spaces.

(Joint work with Andrea Clementi, University of Geneve, Pierluigi Pierini, University of Rome "La Sapienza" and Mauro Felici, Italian Institute of High Mathematics)

Von Neumann's Universal Constructor

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The universal constructor of John von Neumann is an extension of the logical concept of universal computing machine. In the cellular environment proposed by von Neumann both computing and constructive universality can be achieved. Von Neumann proved that in his cellular lattice both a Turing machine and a machine capable of producing any other cell assembly, when fed with a suitable program, can be embedded. He called the latter machine a "universal constructor" and showed that, when provided with a program containing its own description, this is capable of self-reproducing. Self-reproduction takes place through two different processes: during the first the program is interpreted so as to generate a copy of the constructor; during the second a copy of the program is produced and attached to the copy of the

constructor. Von Neumann's conceptual extension is relevant from a bio-theoretical standpoint as it affords the logical basis necessary to define the conditions under which a system is capable of self-reproducing. Unfortunately, because of the rigid determinism governing the automaton evolution and the lack of a minimum of fault-tolerance, von Neumann's automata are not good models of livings beings, contrary to what would be expected.

The structure of a cellular automaton lattice makes it possible to arrange several computations in parallel. However, since von Neumann's proof about computing universality of his automata consisted of the effective implementation of a Turing machine, the information processing proposed in his book "Theory of Self-Reproducing Automata" is not efficient from a computational standpoint. To achieve relative efficiency in parallel computation with cellular automaton lattices, new transition rules and more extended sets of cell states can be introduced so as to include signal crossing and binary representation in order to perform logical and arithmetical operation while preserving the universal constructive capability. In a cellular lattice governed by such a transition rule it is possible to implement algorithms for matrix calculus (matrix sum, product, inversion), solve linear systems, perform prefix and real nummber arithmetical computations, sort arrays, implement functions by power series and so on. The efficiency of the computations is relatively low since the average number of excited cells (or active computing elements) is small compared to the quiescent cells of the lattice. Interestingly, by exploiting the constructive properties of von Neumann's automata, a specific cell assembly for each computation can be built. In fact, one of the most embarrassing aspects of parallel processing is that the optimal architecture for solving a given problem depends critically upon the kind of problem. In the more general model different architectures for parallel computations can be built by governing the first phase of the lattice evolution. Moreover it is conceivable that, in a lattice possessing efficient computational and constructive capabilities, the main process is capable to direct many constructive parallel processes and exploit these for performing new parallel computations. In this kind of process it is possible to allocate (through construction) and de-allocate (through destruction) several parallel sub-processes at the same time.

Parallel Chip Firing on Digraphs

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Given some multidigraph, a *state* is any distribution of some chips on its vertices. Now we transform this initial state step by step. Every vertex checks whether it is able to send one chip through every outgoing arc. If it can, it does, otherwise it does not send any chip. All vertices check and send in parallel. Finally, at every vertex all incoming chips are added to the remaining chips. This transformation on the set of states is iterated.

If the digraph and the total number of chips are finite, then we finally arrive at

some periodic configuration. It is investigated how these periodic configurations and the periods look, depending on the digraph and the total number of chips. There is a sharp contrast in the behavior for Eulerian digraphs (where the in-degree of each vertex equals its out-degree) and non-Eulerian digraphs.

A programming environment for cellular computers

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I present a programming environment for very large-scale cellular computers that I have implemented for the GAPP SIMD processor. It includess programming tools for the design, graphical visualization, implementation and debugging of cellular programs within a software architecture similar to that found in VLSI design systems. My system has four levels of specification and programming language, an interactive editing tool and a shared database that can store various aspects of a cellular design. I present a description of the four constituent levels, each of which corresponds to a particular type of cellular configuration: seed, organ, "moving" or dynamic and program configurations. By way of programming examples, I show how these constrained-based, level-specific languages facilitate the creation and manipulation of cellular configurations. On the basis of the tools developed for and with our programming environment, significant parts of the von Neumann's 29-state cellular automaton and the automaton's transition rule have been implemented on the GAPP processor.

Self-similarity and automaticity of orbits of a class of cellular automata

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Report on a joint work with J.-P. Allouche, F. v. Haeseler, H.-O. Peitgen.

The orbits of finite initial configurations w.r.t. p^k-state linear cellular automata are p-automatic double sequences for a prime number p.

Pascal's triangle modulo a composite number (not a prime power) is not a k-automatic double sequence for any k.

The spacial distribution of states and blocks in the orbit of the initial configuration with only one nontrivial state w.r.t. equivariant m-Fermat cellular automata is described by a self-similar measure on its rescaled evolution set.

De Bruijn Automata and the edge of chaos

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Every linear CA gives rise to a regular language, the set of all finite words that appear as factors of the infinite patterns occurring after one time step. We discuss the size of the minimal automata recognizing these languages. As it turns out, there is an alternative normal form for regular languages associated with CAs, so-called Fischer automata (transitive, deterministic, reduced semiautomata). We show that a Fischer automaton is a strongly connected component in the minimal DFA. We exhibit a fairly large class of binary CAs where the corresponding Fischer automaton has maximal exponential size. All these automata have Hamming distance 1 to a permutation automaton.

Multifractal Formalism for Sofic Measures

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Space-time pattern of linear cellular automata exhibits fractal pattern. To characterize this fractals, we define dimension spectrum $H(\delta)$ by Hausdorff dimension of the set $\{y : \text{dimension of space pattern at height } y \text{ is } \delta\}$. $H(\delta)$ is calculated through the Legendre transformation of the free energy,

$$\Phi_d(\beta) = \lim_{n\to\infty} \frac{\log \sum_{t<p^n} \left(\#\{i : a_i^t \neq 0\}\right)^\beta}{\log p^n}.$$

This relation has been generalized to sofic measures. Let $A_0, \ldots, A_{p-1}$ be nonnegative matrices, v be a nonnegative row vector and u be the nonnegative right eigenvector of $A_0 + \cdots + A_{p-1}$. Sofic measure μ of a cylider set $[y_1 \ldots y_n]$ is given by

$$\mu([y_1 \ldots y_n]) = \frac{v A_{y_1} \ldots A_{y_n} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}}{\lambda^n v u},$$

where λ is the Frobenius eigenvalue of $A_0 + \cdots + A_{p-1}$.

Let the singularity spectrum $f(\alpha)$ be the Hausdorff dimension of the set

$$\{y : \lim_{n\to\infty} \frac{\log \mu([y_1 \ldots y_n])}{\log r^{-n}} = \alpha\},$$

and the free energy $\Phi(\beta)$ be $\lim_{n\to\infty} \frac{\log \sum_{y} mu([y_1 \ldots y_n])^\beta}{log r^{-n}}$. We have established the multifractal formula between the singularity spectrum and the free energy: $f(\alpha) =$

$\inf_{\beta \geq 0} (\alpha\beta - \Phi(\beta))$, with α between $\alpha_{min} = \inf\{\alpha : f(\alpha) > 0\}$ and $\alpha_0 = \inf\{\alpha' : f(\alpha') = \max_\alpha f(\alpha)\}$.

This multifractal formalism has been applied to obtain the Hausdorff dimension of self-affine sets as $\max(\delta + H(\delta)) = \Phi(\eta)$, where η is the ratio of the log of vertical contraction rate to that of horizontal one.

Thermodynamic Behavior of One-Dimensional Reversible Cellular Automata

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In this paper, we consider a family of one-dimensional cellular automata as dynamical systems on which statistical mechanics is constructed. To do this, models need to have reversibility and an additive conserved quantity that can be regarded as energy. The former property is automatically satisfied by using the following time-reversal invariant second-order rules,

$$\sigma_i^{t+1} = f(\sigma_{i-1}^t, \sigma_i^t, \sigma_{i+1}^t) - \sigma_i^{t+1} \pmod 2$$

where $\sigma_i^t \in \{0, 1\}$. The reversibility of the rules is evident for any function $f : \{0, 1\}^3 \to \{0, 1\}$ by construction of the rules. For the latter property derived is a necessary and sufficient condition for a rule in this family to possess an additive conserved quantity of the form

$$\Phi^t = \sum_i F(\sigma_i^t, \sigma_i^{t-1}, \cdots, \sigma_{i+\alpha}^t, \sigma_{i+\alpha}^{t-1})$$

where α is a given positive integer, under the periodic boundary condition. If such a conserved quantity exists, we can define temperature for the system by regarding the conserved quantity as a Hamiltonian and applying the standard Gibbs formalism of statistical mechanics. Moreover, once the temperature is defined, we can simulate heat conduction phenomena by attaching heat baths at both ends of the system. However, if not only the sum Φ but also density F itself is conserved, heat conduction does not occur. Thus we need rules which possess only the additive conserved quantities with no density conserved by itself. Seven such rules are found in this family in the case of $\alpha = 1$. Numerical simulations have been executed for the seven rules and various properties such as formation of global temperature gradient, Fourier's law of heat conduction, the Green-Kubo formula for thermal conductivity, and realization of local equilibrium have been examined. The results obtained show that some rules exhibit diffusive behavior and others exhibit ballistic behavior. To investigate possible connection between the number of additive conserved quantities and the thermodynamic behavior, a Boltzmann-type approximation is introduced and compared with the simulation results. It is concluded that the diffusive behavior is generic in the systems with only one additive conserved quantity but is never observed in the systems with more than one additive conserved quantity.

Self-reproduction in cellular automata: software and hardware realizations

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Our work at the Logic Systems Laboratory at the EPFL in Lausanne, Switzerland, involves the study of the possibility of applying rules of biology (such as healing, reproduction, and evolution) to digital hardware systems. In particular, we use CA to study the phenomenon of self-reproduction. In the tradition of Von Neumann's universal constructor and Langton's loop, we want to study cellular "organisms" capable of (universal) computation and at the same time of self-reproduction. One approach (Perrier & Zahnd) involves adding a (universal) Turing machine to Langton's loop, and thus couple the loop's capability for self-reproduction and the TM's capability for computation. To date, we have been able to add a non-universal TM and achieve self-reproduction of the machine thus obtained. Another approach (Tempesti) is to create a new machine, somewhat similar to Langton's loop in its basic structure, capable not only of self-reproduction but also of executing a program, stored in the circulating loop alongside the instructions directing the self-reproduction. Having achieved this, we are now planning to add a universal TM to the system. The third approach is to abandon, at least in part, the traditional definition of CA to create a new cellular structure, whose basic principles are derived in equal parts from biology, FPGA technology, and CA. This new structure is capable to implement any digital logic circuit and was especially designed to include self-repair (i.e. healing in biological terms) and self-reproduction as intrinsic features. This approach, a collective pursuit of the LSL, has in fact been implemented in hardware as an array of cells, presented as a demo at the seminar.

Real Time One Way Cellular Automata

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A one way cellular automata (OCA) is a CA where the communication is restricted to one way. We are interested in OCA as language recognizer, in particular in the lowest class of complexity, the class of languages recognized in real time. We present examples of languages that are not real time OCA languages using two approches based on counting arguments.

Surjectivity of Cellular Automata Rules

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A number of different properties of cellular automata which are equivalent to surjectivity are presented and discussed. A matrix technique based on the de Bruijn diagram is introduced, and it is shown that each CA rule defines a semi-group of matricies such that the rule is surjective if and only if this semi-group does not contain the zero matrix. Another approach to surjectivity based on the subset diagram for a CA is also discussed, and certain replacement diagrams are defined which generate sequences counting numbers of pre-images of finite strings.

Three Remarks on Additive Cellular Automata

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Three aspects of the theory of additive cellular automata are discussed: 1) A method for computing predecessor states for any additive rule is given; 2) It is shown that an additive rule is injective if and only if a certain complex integral is zero; 3) The obstruction to additivity for cellular automata is defined as a map from the cartesian product of the state space to the state space, and some properties of this map are derived.

On classes of transition functions satisfying a Frobenius law

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The investigation of powers even only of the 256 elementary cellular automaton rules is complicated. Therefore we restrict ourselves to a case where in addition some algebraic structure is imposed upon the set of states and on the set of transition functions to be considered. More specifically we speak of an algebraic automaton, if the set of "global states" is a group $(G, +)$ and a transition function has to be a group homomorphism $f \in \mathrm{End}(G)$. If A is a subring of $\mathrm{End}(G)$ then G is a module over A with function application $(a, g) \mapsto a(g)$ as external multiplication. A is called Frobenian, if for all $a, b \in A$ and all $n \in \mathsf{N}$ it holds, that $(a + b)^n = a^n + b^n$.

It turns out that this is a strong condition which leads to a lot of structure in such rings. For example for all $a \in A : a^4 = a^5 = a^6 = \cdots$ holds. Under the additional requirement, that all elements of A are idempotent, the following is true: Finite Frobenian rings are exactly the set rings where the domain is the power set of an arbitrary set and addition and multiplication are in fact symmetric difference and

intersection. In the infinite case Frobenian rings can at least always be embedded into a power set ring. From set rings the concepts of a partial order on the domain and that of singletons can be carried over to Frobenian rings. If A has the property that each element a is uniquely determined by the set of singletons $s \leq a$, then each module over A is isomorphic to the subring of all finite subsets of a power set ring.

In order to apply these concepts to cellular automata it is required that the set of states and hence also the set of all configurations are groups. Unfortunately the requirement of a ring of (global) transition functions to be Frobenian probably is too restrictive. If for example G is a finite set of transition functions such that the generated ring $H = \langle G \rangle$ is Frobenian, then H also is finite! Hence only a few non trivial examples are known.

Fault tolerant solutions to the firing squad synchronization problem

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We are interested in particular instances of the well known Firing Squad Synchronization Problem in which some cells are defective.

We call p-faulty n-line a line of n cells in which there are p "regions". The i-th region is made of n_i connected non-faulty cells and m_i connected defective ones, and the line is ended with n_{p+1} working automata:

$$n = \sum_{i=1}^{i=p} (n_i + m_i) \; + \; n_{p+1}$$

Umeo proved that given a p-faulty n-line, where p is an unknown integer and $\forall i \in [1, p]$, $n_i \geq m_i$ and $n_i + m_i \geq p - i$, it is possible to construct a fault tolerant $(2n - 2 + p)$-steps nearly optimum time firing squad algorithm.

We tried to *reverse* Umeo's conditions and proved that:

Theorem 3 *Given a p-faulty n-line, where p is a fixed integer and $\forall i \in [1, p]$, $n_i \leq m_i$, it is possible to construct a fault tolerant algorithm which synchronize in $2n + \sum_{i=1}^{i=p}(m_i - n_i)$ units of time, if:*

$$\forall i \in [1, p-1],\ n_{i+1} \geq m_i - n_i \text{ and } n_{p+1} \geq 2m_p - n_p$$

We are able to construct a *mixed* solution in which for each region we have (as desired) more faulty or more non-faulty cells, thus we proved that :

Theorem 4 *Given a p-faulty n-line, where p is a fixed integer, it is possible to construct a fault tolerant algorithm which synchronize in $2n + \sum_{i=1}^{i=p} |m_i - n_i|$ units of time, if :*

$$\forall i \in [1, p],\ n_{i+1} \geq |m_i - n_i| \text{ and } n_{p+1} \geq max(n_p, m_p) + |m_p - n_p|$$

We show how to construct a fault tolerant solution even if we don't know p the number of faulty regions.