Program

Monday

- 7:30 - 8:45 Breakfast
- 9:00 **Colin Defant**

  Three Topics in Pattern Avoidance

  This talk will survey three topics related to pattern avoidance, each motivated by an interesting question:

  (a) Given a class C of objects and a set P of patterns from C, what is the smallest size of an object in C that contains all of the patterns in P? We will focus on recent developments in which C is either a class of words or a class of trees.

  (b) How does permutation pattern avoidance interact with the group structure of the symmetric group? The first part of this topic will concern pattern-avoiding permutations with particular cycle types; the second part will concern permutations whose powers are required to avoid a pattern.

  (c) If your socks come out of the laundry all mixed up, how should you sort them? We will discuss a novel foot-sorting algorithm that uses feet to attempt to sort a sock ordering; one can view this algorithm as an analogue of Knuth's stack-sorting algorithm for set partitions.

- 10:00 Coffee Break
- 10:20 **Letong (Carina) Hong**

  Length-Four Pattern Avoidance in Inversion Sequences

  Pattern avoidance for permutations is a robust and well-established branch of enumerative combinatorics since the systematic study of Simion and Schmidt in 1985 ([1]). We say two patterns are Wilf equivalent if the number of sequences avoiding them, respectively, is equal. An inversion sequences of length n is an integer sequence $e_1, ..., e_n$ with $0 \leq e_i < i$ for all i. This paper discusses how in recent work all Wilf equivalence classes of pattern-avoiding inversion sequences of all length-4 patterns are classified. This is based on joint work with Rupert Li, and one originally unsolved case is tackled independently by Chern, Fu, and Lin; and Mansour.

- 11:05 **Anders Claesson**

  On the problem of Hertzsprung and similar problems

  Drawing on a problem posed by Hertzsprung in 1887 (sometimes called the n-kings problem), we say that a permutation w contains the Hertzsprung pattern u if there is factor of w that differ only by a constant from u in the sense that there is a factor w(d+1)w(d+2)...w(d+k) such that w(d+1)-u(1) = ... = w(d+k)-u(k). Using a combination of the Goulden-Jackson cluster method and the transfer-matrix method we determine the joint distribution of occurrences of any set of Hertzsprung patterns, thus substantially generalizing earlier results by Jackson et al. on the distribution of ascending and descending runs in permutations. We apply our results to the problem of counting permutations up to pattern-replacement equivalences, and using pattern-rewriting systems—a new formalism similar to the much studied string-rewriting systems—we solve a couple of open problems raised by Linton et al. in 2012

- 12:15 Lunch
- 15:00 Coffee and Cake
- 15:45 **Sergi Elizalde**

  Walks in simplices, cylindric tableaux, and asymmetric exclusion processes

  We describe bijections between three classes of combinatorial objects that have appeared in different contexts: lattice walks in simplicial regions as introduced by Mortimer–Prellberg (in a previous Dagstuhl seminar), standard cylindric tableaux as introduced by Gessel–Krattenthaler and Postnikov, and sequences of states in the totally asymmetric simple exclusion process. Our perspective gives new insights into these objects,

  providing a vehicle to translate enumerative results and certain symmetries from one setting to another.

  As an example, we use a cylindric analogue of the Robinson–Schensted correspondence to give an alternative bijective proof of a recent result of Courtiel, Elvey Price and Marcovici relating forward and backward walks in simplices.
- 16:30 **Erik Slivken**

  Scaling Limits of Restricted Permutations

  Suppose we take a large permutation that is chosen uniformly at random and conditioned to satisfy some restriction. What does this permutation look like? The answer depends on the choice of restriction and how one decides to scale the permutation. We introduce a few scaling limits that prove useful in answering this type of question for a variety of restrictions, especially in the case of pattern-avoiding permutations. We will explore what various scaling limits say about these objects and some associated statistics (like the number of fixed points of the permutation).
- 17:15 **Open Problem Session I**

- 18:00 Dinner.

Tuesday

- 7:30 - 8:45 Breakfast
- 9:00 **Péter Csikvári**

  On complex roots of the independence polynomial

  The independence polynomial of a graph is the generating polynomial of all its independent sets. Formally, given a graph G, its independence polynomial $Z_G(\lambda)$ is given by $\sum_I \lambda^{|I|}$, where the sum is over all independent sets I of G. The independence polynomial has been an important object of study in both combinatorics, statistical physics and computer science. In particular, the algorithmic problem of estimating $Z_G(\lambda)$ for a fixed positive λ on an input graph G is a natural generalization of the problem of counting independent sets, and its study has led to some of the most striking connections between computational complexity and the theory of phase transitions. More surprisingly, the independence polynomial for negative and complex values of λ also turns out to be related to problems in statistical physics and combinatorics. In particular, the locations of the complex roots of the independence polynomial of bounded degree graphs turn out to be very closely related to the Lovász local lemma, and also to the questions in the computational complexity of counting. In this talk we give new geometric criteria for establishing zero-free regions as well as for carrying out semi-rigorous numerical explorations. We then provide several examples of the (rigorous) use of these criteria, by establishing new zero-free regions. Joint work with Ferenc Bencs, Piyush Srivastava and Jan Vondrák.
- 10:00 Coffee Break
- 10:20 **Justin Troyka**

  Pattern-avoiding affine permutations

  An affine permutation of size n is a bijection p from the integers to the integers satisfying certain properties, including that p(i+n) = p(i)+n for all i. The affine permutations of size n have been much studied as a Coxeter group, but our perspective is pattern avoidance. To have a finite set to count, we can require also that |p(i) - i| < n for each i; such p we call a bounded affine permutation. We give the asymptotic number of bounded affine permutations of size n that avoid k...1; in particular, they have the same growth rate as the ordinary permutations avoiding k...1. We also show several results about affine permutation classes with the property that every element is a shift of the infinite direct sum of an ordinary permutation, from which we obtain exact enumerations for several affine permutation classes. This talk covers joint work with Neal Madras, published in 2021 in Discrete Math. Theor. Comput. Sci. and in Ann. Comb. Our work, especially the boundedness condition, is motivated by the fruitful concept of periodic boundary conditions in statistical physics.

- 11:05 **Gökhan Yildirim**

  Generating Tree Method and Pattern Avoiding Inversion Sequences

  An *inversion sequence* of length n is an integer sequence $e = e_1 \cdots e_n$ such that $0 \le e_i < i$ for each $0 \le i \le n$. We use I_n to denote the set of inversion sequences of length n. Any word τ of length k over the alphabet $[k] := \{0, 1, \cdots, k-1\}$ is called a pattern. For a given pattern τ, we use $I_n(\tau)$ to denote the set of all τ-avoiding inversion sequences of length n.

  Pattern-avoiding inversion sequences were systematically studied first by Mansour and Shattuck (2015) for the patterns of length three with non-repeating letters and by Corteel et al. (2016) for repeating and non-repeating letters. Since then, researchers have obtained several interesting results for these combinatorial objects.

  We provide an algorithmic approach based on generating trees for enumerating the pattern-avoiding inversion sequences. First, by using this algorithmic approach, we determine the generating trees for many pattern classes such as $I_n(100), I_n(011, 201), I_n(021, 0112), \cdots$. Then we obtain enumerating formulas for them through generating functions and the kernel method.

  The talk is based on a joint work with Ilias S. Kotsireas (Wilfrid Laurier University) and Toufik Mansour (University of Haifa).

- 11:45 **Open Problem Session II**
- 12:15 Lunch
- 15:00 Coffee and Cake
- 15:45 **Jessica Striker**

  Two new bijections on six-vertex configurations

  The six-vertex model is an exactly solvable model in statistical mechanics that has been widely studied by both physicists and combinatorialists for its many lovely properties.

  In this talk in two parts, we first describe joint work with Daoji Huang giving a bijection between alternating sign matrices (a combinatorial manifestation of six-vertex configurations) and totally symmetric self-complementary plane partitions in the reduced, 1432-avoiding case. Finding such a bijection in full generality has been an open problem for nearly 40 years; it will be interesting to see whether this new sub-bijection may lead to further (positive or negative) results on a full bijection.

We then introduce a symmetrized version of the six-vertex model that adapts nicely from the square lattice to arbitrary 4-regular graphs embedded in a disk. By modifying certain vertex configurations, we transform these to nearly planar bipartite graphs that have regions corresponding to dimer covers of hexagonal lattices. Our underlying motivation and main result is a bijection between equivalence classes of these graphs and 4-row tableaux in such a way that promotion corresponds to rotation, yielding a web basis for SL_4x. This is joint work with Christian Gaetz, Oliver Pechenik, Stephan Pfannerer, and Joshua Swanson.

- 16:30 **Sylvie Corteel**

  Permutations and exclusion processes

  We will review results on the combinatorics of the exclusion process which is a classical model in statistical physics. We will explain why permutations and the pattern 31-2 appear when we study the exclusion process with open boundaries. We will then propose a series of open problems on the combinatorics of more general processes.

- 17:15 **David Bevan**

  Mesh pattern occurrence in random permutations

  If p is a mesh pattern, let $\kappa(p) = 1 - \lim_{n\to\infty} |\mathrm{Av}_n(p)|/n!$ be the asymptotic probability that a permutation contains an occurrence of p (if the limit exists). We call $\kappa(p)$ the *likelihood* of p. We investigate the values of the likelihood of a variety of patterns, determining $\kappa(p)$ whenever p is vincular (or covincular) or is a bivincular *frame* or *ladder*. All these likelihoods are rational.

  Other bivincular patterns have irrational likelihoods. The likelihood of a small ascent or a small descent (the intervals of size two) is $1 - e^{-1} \approx 0.63212$, a result which has been known since the 1940s. By generalising a proof of this using the Chen–Stein method, we establish the likelihoods of a wide variety of bivincular patterns, often yielding values expressed using special functions. However, a complete analysis of bivincular patterns currently seems to be beyond reach.

  Here are three open questions about the set of likelihood values:

  (a) Is there a pattern whose likelihood is rational, but is not the reciprocal of a product of factorials?

  (b) Are the small ascent and the small descent the only patterns with likelihood strictly between $\frac{1}{2}$ and 1?

  (c) Is $(1 - e^{-1})^2 \approx 0.39958$ the largest likelihood less than $\frac{1}{2}$?

  This is joint work with Jason Smith.

- 18:00 Dinner

Wednesday

- 7:30 - 8:45 Breakfast
- 9:00 **István Miklós**

  Computational complexity of counting and sampling

  Whenever we can ask if a certain mathematical object with prescribed properties exists, we can also ask how many such objects exist and how to generate a random one. We can also talk about the computational complexity of these counting and sampling problems, that is, how the running time of computer programs solving these problems increases with the input size. It turns out that counting and sampling are equally hard for a large class of computational problems. Equal hardness also frequently holds for estimating weighted sums and sampling from a distribution proportional to these weights. For example, in statistical physics, these problems are sampling from the Boltzmann distribution and estimating the partition function. In this talk, we give an overview of sampling and counting complexity, then we will focus on the most powerful approach to sampling, the Markov chain Monte Carlo method.
- 10:00 Coffee Break
- 10:20 **Benjamin Berendsohn**

  Saturation for permutation matrices

  A 0-1 matrix M contains a 0-1 matrix pattern P if we can obtain P from M by deleting rows and/or columns and turning arbitrary 1-entries into 0s. The saturation function sat(P, n) for a 0-1 matrix pattern P indicates the minimum number of 1s in an $n \times n$ 0-1 matrix that does not contain P, but where changing any 0-entry into a 1-entry creates an occurrence of P.

  Saturation for 0-1 matrices was introduced by Brualdi and Cao [arXiv 2020]. Fulek and Keszegh [SIAM J. Discret. Math. 2021] started a systematic study, and showed that each pattern has a saturation function that is either bounded or linear. They found large classes of patterns with linear saturation function, but only a single pattern with bounded saturation function.

  Subsequently, Geneson [Electron. J. Comb. 2021] showed that almost all *permutation matrices* have bounded saturation functions. In this talk, we outline how to complete the classification of permutation matrices using a construction based on oscillations in indecomposable permutation matrices.
- 11:05 **László Kozma**

  Pattern avoidance: algorithmic connections

Already from the beginnings of the field, pattern-avoidance has been studied in tandem with algorithmic applications. For instance, Knuth's study of permutation-patterns was motivated by connections to stack-sorting. In this talk I will survey results from the last few years on two related lines of work:

(1) the study of algorithms, in various models of computation, for detecting, counting, or enumerating patterns, and
(2) the study of pattern-avoidance as a source of algorithmic "easiness": how pattern-avoidance of the input affects the complexity of seemingly unrelated algorithmic tasks.

- 12:00 Group Photo
- 12:15 Lunch
- Afternoon Excursion
- 18:00 Dinner.

Thursday

- 7:30 - 8:45 Breakfast
- 9:00 **Gábor Tardos**

  Extremal theory of vertex and edge ordered graphs

  Turán-type extremal graph theory has been generalized in many directions. In this talk I will survey the extremal theories of vertex and edge ordered graphs. In the vertex ordered case the vertices of the host graph are linearly ordered and we only forbid a subgraph with a specified vertex order. As in the classical extremal graph theory, we are still looking for the maximal number of edges in a host graph on n vertices avoiding the forbidden subgraph. The analogous extremal theory for edge ordered graphs was introduced recently in a paper of Gerbner, Methuku, Nagy, Pálvölgyi, T., Vizer (2023). In contrast, the vertex ordered theory has a much richer history going back to related extremal matrix problems studied by Füredi and Hajnal in 1992.

  Both theories are rich in specific results: the extremal functions of some small forbidden ordered graphs. Some of these results found applications in combinatorial geometry. Other specific forbidden patterns lead to interesting open problems.

  Another direction is to find analogues of general results from classical extremal graph theory. The analogues of the Erdős-Stone-Simonovits theorem has been found in both theories: the key is to find the "correct" version of the chromatic number that applies. For vertex ordered graphs, this is the simple notion of interval chromatic number, for edge ordered graphs, however, the corresponding notion is surprisingly rich.

In the classical (unordered) theory we have a very simple dichotomy: forests have linear extremal functions, while other graphs have far-from-linear extremal functions. The search for the analogue of this simple observation yielded several nice results, conjectures and open problems about vertex and edge ordered graphs.

- 10:00 Coffee Break
- 10:20 **Anthony Labarre**

  Sorting Genomes by Prefix Double-Cut-and-Joins

  A double cut-and-join (DCJ for short) is an operation that replaces two edges u, v and w, x in a graph with either $\{u, x\}, \{v, w\}\}$ or $\{\{u, w\}, \{v, x\}\}$. This operation is of interest in a biological context, as it generalises several other well-studied mutations that are known to happen in genomes, e.g. (possibly signed) reversals or (block-)transpositions.

  I consider two different graph models for representing genomes — namely, paths and perfect matchings — and study DCJs under the "prefix restriction", which forces one of the cut edges to contain the first element of the genome. The talk focuses on sorting problems using these operations, which is a popular approach to reconstructing evolutionary scenarios between species, but also has applications in the field of interconnection network design, from which the prefix restriction originates. I will present some recent results on sorting genomes using variants of prefix DCJs using both models. Namely:

  - new lower bounds on sorting genomes using prefix DCJs or prefix reversals;
  - a polynomial-time algorithm for sorting signed genomes by prefix DCJs; and
  - a 3/2-approximation algorithm for sorting unsigned genomes by prefix DCJs.

  The latter algorithm is the first polynomial-time approximation algorithm with a ratio smaller than 2 for a prefix sorting problem not known to be in P.

- 11:05 **Luca Ferrari**

  Fighting fish and pattern avoiding permutations

  Fighting fish are combinatorial objects recently introduced by Duchi, Guerrini, Rinaldi and Schaeffer. They are polyomino-like objects which can branch out of the plane into independent substructures. The main motivation for considering such structures lies in their remarkable probabilistic properties. In particular, it is known that the average area of fighting fish having semiperimeter n is of order $n^{5/4}$, which is a rather non-standard behaviour. The previously mentioned authors have discovered a lot of interesting combinatorics related to fighting fish. In particular, they have shown that the number of fighting fish of semiperimeter $n+1$ is given by $\frac{2}{(n+1)(2n+1)}\binom{3n}{n}$, which is the same as the number of (West)-two-stack sortable permutations of size n. This result spurred much research to better understand the relationships between these objects (and others also counted by the same sequence). Wenjie Fang was able to describe a bijection between two-stack-sortable permutations and fighting fish which accounts for the above enumerative result (and also preserves a lot of other statistics). However, his bijection is recursive, and no direct bijection is known yet. In my talk I will present a general construction which maps any permutation to a certain labelled tree, which in turn encodes a unique fighting fish. Such a construction is not bijective in general, but it becomes a bijection when restricted to two-stack-sortable permutations, thus defining the (almost) direct bijection that was missing. As a matter of fact, this construction is equivalent to Fang's one (in other words, it is a more direct version of the recursive procedure of Fang). Our hope would then be to use our construction to understand what the parameter area (on fighting fish) is on permutations, but we have not been successful yet. We have however some partial results, such as a description of those permutations (of fixed size) whose associated fighting fish has minimum area.

The sequence enumerating fighting fish with respect to semiperimeter also counts another class of permutations, namely those avoiding the two vincular patterns 3-1-4-2 and 2-41-3. Such permutations have been investigated by Claesson, Kitaev and Steingrimsson, in particular they provide a bijection with so-called $\beta(1,0)$-trees. These are trees whose recursive structure encodes somehow more naturally the recursive structures of fighting fish. This leads us to think that this class of permutations could be more useful to have a better combinatorial understanding of the area of fighting fish.

- 12:15 Lunch
- 15:00 Coffee and Cake
- 15:45 **Mathilde Bouvel**

  Non-uniform permutations biased according to their records

  Abstract: In the average-case analysis of algorithms working on arrays of numbers (modeled by permutations), the uniform distribution on the set of possible inputs is usually assumed. However, the actual data on which these algorithms are used is rarely uniform, and often displays a bias towards "sortedness". In this talk, we present a non-uniform distribution

on permutations, which favors their records (a.k.a. left-to-right maxima). We describe the behavior (on average and in distribution) of some classical permutation statistics in this model, some of which with applications to the analysis of algorithms. We also describe the "typical shape" of permutations in our model, by means of their (deterministic) permuton limit.

This is joint work with Nicolas Auger, Cyril Nicaud and Carine Pivoteau.

- 16:30 **Radu Curticapean**

  The complexity of computing immanants

  Immanants are matrix functions that generalize determinants and permanents. Given an irreducible character χ_λ of S_n for some partition λ of n, the immanant associated with λ is a sum-product over permutations π in S_n, much like the determinant, but with $\chi_\lambda(n)$ playing the role of $sgn(\pi)$.

  Hartmann showed in 1985 that immanants can be evaluated in polynomial time for sign-ish characters. More precisely, for a partition λ of n with s parts, let $b(\lambda) := n - s$ count the boxes to the right of the first column in the Young diagram of λ. The immanant associated with λ can be evaluated in $n^{O(b(\lambda))}$ time.

  Since this initial result, complementing hardness results have been obtained for several families of immanants derived from partitions with unbounded $b(\lambda)$. This includes permanents, immanants associated with hook characters, and other classes. In this talk, we complete the picture of hard immanant families: Under a standard assumption from parameterized complexity, we rule out polynomial-time algorithms for well-behaved immanant families with unbounded $b(\lambda)$. For immanant families in which $b(\lambda)$ even grows polynomially, we establish hardness for $\#P$ and VNP. (See arXiv:2102.04340.)

- 17:15 **Henning Úlfarsson**

  Exploring Permutation Classes with TileScope

  In the world of combinatorics, there are numerous sets of objects that are in a one-to-one correspondence with sets of permutations possessing specific properties, commonly characterized by pattern avoidance. This talk will delve into the TileScope algorithm and demonstrate its usefulness in comprehending permutation sets avoiding a finite list of patterns. Additionally, we will examine the algorithm's output, including polynomial counting formulas, systems of equations, uniform random generation, and more. Finally, we will highlight the algorithm's ability to discover bijections automatically, examine the atomicity of permutation classes, and preview planned future advancements. Visit permpal.com to see successful applications of the algorithm in action.

This is joint work with Michael Albert, Christian Bean, Anders Claesson, Émile Nadeau, and Jay Pantone.

- 18:00 Dinner.

Friday

- 7:30 - 8:45 Breakfast
- 9:15 **Vit Jelinek**

  What makes permutation patterns hard to match?

  Permutation Pattern Matching (or PPM) is a fundamental decision problem in the study of permutations. Its input is a pair permutations P (the "pattern") and T (the "text"), and the goal is to determine whether P is contained in T. While PPM is NP-hard on general inputs, it often becomes tractable when P or T are restricted to a proper hereditary permutation class.

  In my talk, I will present some results and conjectures that attempt to describe the boundary between tractable and hard cases of such restrictions of PPM and other related problems. Specifically, I will focus on the relationships between these three topics: computational complexity of PPM restricted to a given permutation class; structural properties of a permutation class, such as the presence of large grid-like substructures; and the growth of width parameters, like tree-width or grid-width, within a given permutation class.

- 10:00 Coffee Break
- 10:20 **Natasha Blitvic**

  Combinatorial Moment Sequences

  Take your favorite integer sequence. Is this sequence a sequence of moments of some probability measure on the real line? We will look at a number of interesting examples (some proven, others merely conjectured) of moment sequences in combinatorics. We will consider ways in which this positivity may be expected (or surprising!), the methods of proving it, and the consequences of having it. The problems we will consider will be very simple to formulate, but will take us up to the very edge of current knowledge in combinatorics, “classical" probability, and noncommutative probability.

- 12:15 Lunch