Mathematical models developed to describe the trajectories of ”swarming” particles that interact through decentralized forces have shed light on the mechanisms of collective dynamics and selforganization. Besides coupling through direct pairwise forces, individuals can also interact via the medium in which they swim. We derive from first principles a three-dimensional theory of particle swarming that explicitly includes particle-fluid coupling and distinguish between swimmerinduced and interaction-induced flows. The collective dynamics depends on whether particles move in “clear uids,” where they have full knowledge of their surroundings or in “opaque fluids,” where they control their velocities only in relation to the local fluid flow. We show that short-ranged particle-particle interactions can lead to much longer-ranged fluid-mediated hydrodynamic forces, effectively amplifying the range between which particles interact. This induced fluid flow, along with self-generated ows, can conspire to profoundly affect swarm morphology, kinetically stabilizing or destabilizing swarm configurations. Depending upon the interaction potential, the mechanism of force-free swimming (e.g., pushers or pullers), and the degree of fluid opaqueness, we find a number of new collective patterns including flocks with prolate or oblate shapes, recirculating peloton-like structures, and jet-like fluid flows that entrain particles in mill-like structures. Our results reveal how fluid-mediated interactions influence the self-organization, mobility and stability of threedimensional swarms and suggest how they might be used to kinetically control their collective behavior.