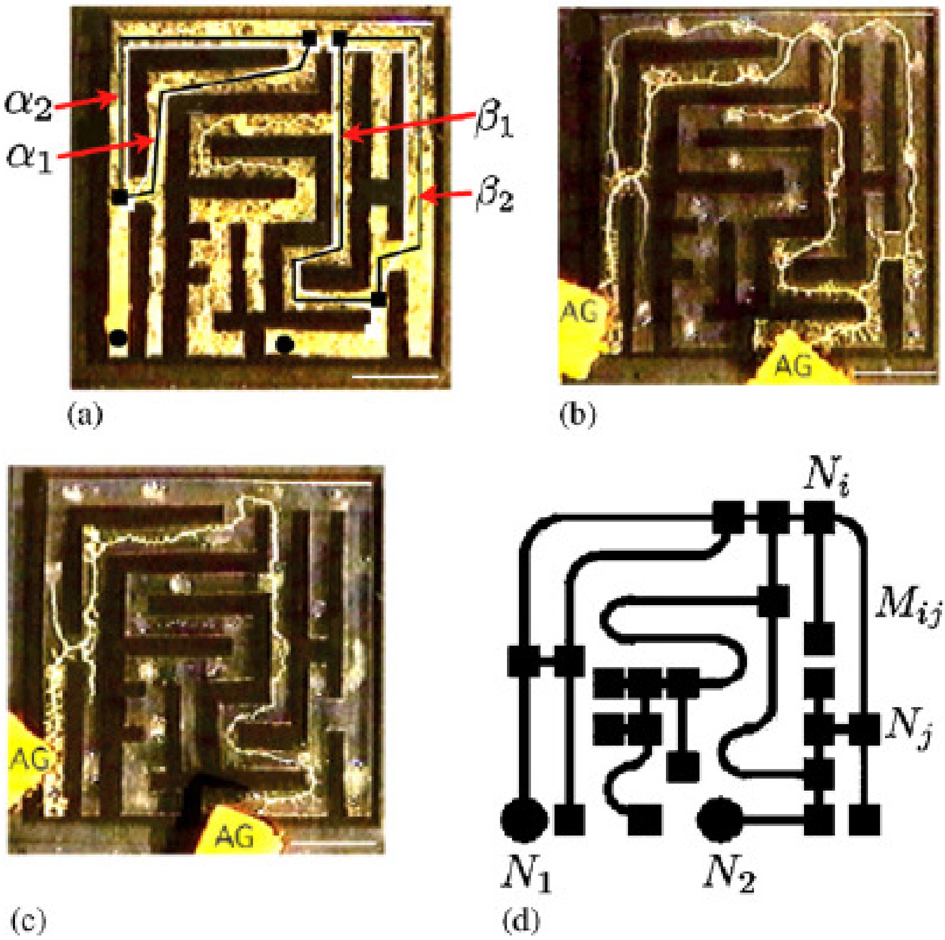

Figure 1: The experiment in [12] (reprinted from there): (a) shows the maze uniformly covered by Physarum; yellow color indicates presence of Physarum. Food (oatmeal) is provided at the locations labeled AG. After a while the mold retracts to the shortest path connecting the food sources as shown in (b) and (c). (d) shows the underlying abstract graph. The video [15] shows the experiment.

Natural Algorithms
Physarum Can Compute Shortest Paths

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The talk is based on joint work [4, 3] with Luca Becchetti, Vincenzo Bonifaci (Rome), Girish Varma (TIFR), Michael Dirnberger (MPI-INF), and Andreas Karrenbauer (MPI-INF).

Nature computes; birds flock and the slime mold *Physarum Polycephalum* is apparently able to solve shortest path problems and to construct good Steiner networks. Nakagaki, Yamada, and Tóth [12] report about the following experiment; see Figure 1. They built a maze, covered it by pieces of Physarum (the slime can be cut into pieces which will reunite if brought into vicinity), and then fed the slime with oatmeal at two locations. After a few hours the slime retracted to a path that follows the shortest path in the maze connecting the food sources. The authors report that they repeated the experiment with different mazes; in all experiments, Physarum retracted to the shortest path. There are several videos available on the web that show the mold in action [15].

The paper [13] proposes a mathematical model for the behavior of the slime and argues extensively that the model is adequate. We will not repeat the discussion here but only define the model. Physarum is modeled as an electrical network with time varying resistors. We have a simple undirected graph $G = (N, E)$ with distinguished nodes s_0 and s_1 modeling the food sources. Each edge $e \in E$ has a positive length L_e and a positive diameter $D_e(t)$; L_e is fixed, but $D_e(t)$ is a function of time. The resistance $R_e(t)$ of edge e is $R_e(t) = L_e/D_e(t)$. We force a current of value 1 from s_0 to s_1. Let $Q_e(t)$ be the resulting current over edge $e = (u, v)$, where (u, v) is an arbitrary orientation of e. The diameter of edge e evolves according to the equation

$$\dot{D}_e(t) = |Q_e(t)| - D_e(t), \tag{1}$$

where $\dot{D}_e$ is the derivative of D_e with respect to time. In equilibrium ($\dot{D}_e = 0$ for all e), the flow through any edge is equal to its diameter. In non-equilibrium, the diameter grows (shrinks) if the absolute value of the flow is larger (smaller) than the diameter. In the sequel, we will mostly drop the argument t as is customary in the treatment of dynamical systems.

The model is readily turned into a computer simulation. Tero et al. [13] were the first to perform such simulations. They report that the network always converges to the shortest s_0-s_1 path, i.e., the diameters of the edges on the shortest path converge to one and the diameters on the edges outside the shortest path converge to zero. This holds true for any initial condition. It assumes uniqueness of the shortest path. Miyaji and Ohnishi [10, 9] initiated the analytical investigation of the model. They argued convergence against the shortest path if G is a planar graph and s_0 and s_1 lie on the same face in some embedding of G.

In [4], we proved convergence for all graphs. In [3], we proved convergence of the discretization.

Why should CS care? Physarum is an example of a natural computer, i.e., a computer developed by evolution over millions of years. It apparently can do more than computing shortest paths. In [14] the computational capabilities of Physarum are applied to network design and it is shown in lab and computer experiments that Physarum can compute approximate Steiner trees; see 2 for an example. However, no theoretical explanation is available. We [7, 8] are currently trying to find one. The book [1] and the tutorial [11] contain many illustrative examples of the computational power of this slime mold.

Chazelle [5] advocates the study of natural algorithms; i.e., "algorithms developed by evolution over millions of years", using computer science techniques. Traditionally, the analysis of such algorithms was the domain of biology, systems theory, and physics. Computer science brings new tools. For example, in our analysis, we crucially use the max-flow min-cut theorem.

Natural algorithms can also give inspiration for the development of new combinatorial algorithms. A good example is [6] where electrical flows are essential for an approximation algorithm for undirected network flow.

References

[1] A. Adamatzky. *Physarum Machines: Computers from Slime Mold.* World Scientific Publishing, 2010.

[2] A. Adamatzky and T. Schubert. Schlauschleimer in reichsautobahnen. *Kybernetes*, pages 1050–1071, 2012.

[3] L. Becchetti, V. Bonifaci, M. Dirnberger, A. Karrenbauer, and K. Mehlhorn. Physarum can compute shortest paths: Convergence proofs and complexity bounds. In *ICALP*, volume 7966 of *LNCS*, pages 472–483, 2013.

[4] V. Bonifaci, K. Mehlhorn, and G. Varma. Physarum can compute shortest paths. *Journal of Theoretical Biology*, 309(0):121–133, 2012. A preliminary version of this paper appeared at SODA 2012 (pages 233-240).

[5] B. Chazelle. Natural algorithms. In *SODA*, pages 422–431, 2009.

[6] P. Christiano, J. A. Kelner, A. Madry, D. A. Spielman, and S.-H. Teng. Electrical flows, laplacian systems, and faster approximation of maximum flow in undirected graphs. *CoRR*, abs/1010.2921, 2010. to appear in STOC 2011.

[7] M. Dirnberger, T. Kehl, T. Mehlhorn, K. Mehlhorn, and A. Neumann. Towards an open online repository of p. polycephalum networks and their corresponding graph representations. In *PhysNet 2015*, 2015.

[8] M. Dirnberger, T. Kehl, and A. Neumann. NEFI: Network extraction from images. *Scientic Reports*. to appear.

[9] T. Miyaji and I. Ohnishi. Mathematical analysis to an adaptive network of the plasmodium system. *Hokkaido Mathematical Journal*, 36:445–465, 2007.

[10] T. Miyaji and I. Ohnishi. Physarum can solve the shortest path problem on riemannian surface mathematically rigourously. *International Journal of Pure and Applied Mathematics*, 47:353–369, 2008.

[11] T. Nakagaki, A. Tero, R. Kobayashi, I. Onishi, and T. Miyaji. Computational ability of cells based on cell dynamics and adaptability. *New Generation Computing*, 27:57–81, 2009.

[12] T. Nakagaki, H. Yamada, and A. Tóth. Maze-solving by an amoeboid organism. *Nature*, 407:470, 2000.

[13] A. Tero, R. Kobayashi, and T. Nakagaki. A mathematical model for adaptive transport network in path finding by true slime mold. *Journal of Theoretical Biology*, pages 553–564, 2007.

[14] A. Tero, S. Takagi, T. Saigusa, K. Ito, D. Bebber, M. Fricker, K. Yumiki, R. Kobayashi, and T. Nakagaki. Rules for biologically inspired adaptive network design. *Science*, 327:439–442, 2010.

[15] `http://www.youtube.com/watch?v=tLO2n3YMcXw&t=4m43s`.

Figure 2: Physarum connects major cities on the German map. Reprinted from [2].