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Interactions between Category Theory and Computer Science

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Interactions between Category Theory and Computer Science

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The conference brought together a small group of researchers actively applying category theoretic and formal logical methods to the following topics: programming language semantics, type theories, linear logic, concurrency theory and abstract data type theory. There were 38 participants (from ten countries), of which 33 presented talks. The pleasant surroundings and well-run facilities provided ample opportunities for interaction between participants.

Scientifically speaking, interactions between category theorists and computer scientists are driven by several concerns. Probably the most important is the need to give adequate formal semantics for programming language constructs, and a number of talks addressed this issue. The concepts developed by category theorists for use in computer science have led to new developments in category theory that are of interest in themselves and category theory developments have also led to suggestions for new features in programming languages. Another side of this picture is supplied by logic. One of the key questions in computer science now concerns tools for developing and reasoning about concurrently reacting systems and parallel programming constructs. It may be that an important role in understanding the structure of parallel programs will come from linear logic, a topic which featured in several of the talks. The interaction here is mediated by the search for the proper categorical semantics for both structures.
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On the Reduction of Chocs Bisimulation to $\pi$-calculus
Bisimulation

ROBERTO M. AMADIO

Chocs and $\pi$-calculus are two extensions of CCS where, respectively, processes and channels are transmissible values. In previous work we have proposed a formalization of the notion of bisimulation for Chocs. In this paper we suggest a more effective way to reason about this notion by means of an embedding of Chocs into a richer calculus endowed with a notion of ‘activation’ channel which we christen Chocs. $t$ is the name of a new internal action which is produced by a synchronization on an activation channel, such a synchronization has the effect of forcing the execution of an idle process. In first approximation transitions in Chocs, may be understood as sequences of synchronizations along activation channels followed by an ‘observable’ transition. There is a simple definition of bisimulation for Chocs, which satisfies natural laws and congruence rules, moreover the synchronization trees associated to Chocs, processes are finitely branching. We propose Chocs as an intermediate step towards the definition of a tool for the verification of Chocs bisimulation.

Hopf Algebras and Linear Logic

RICHARD BLUTE

The goal of this work is to define a general construction for specifying models of multiplicative linear logic. We have two goals in mind. First, we would like the models to be nondegenerate, meaning that the two multiplicative connectives are not equated. Second, we would like the construction to be general enough to model the commutative logic, the noncommutative logic of Lambek and Abrusci, and the braided logic defined by the author.

The first goal is achieved by working with categories of vector spaces equipped with an additional topological structure. The topology allows the construction of infinite dimensional reflexive spaces. The second goal is obtained by working with vector spaces which are also representations of Hopf algebras. By varying the Hopf algebra, we can construct symmetric, nonsymmetric and braided monoidal categories. In the last case, we are led to the notion of quantum group.
Full Abstraction for a Shared Variable Parallel Language

STEPHEN BROOKES

We give a trace semantics for a language of shared variable parallel processes and show that the semantics is fully abstract with respect to partial correctness behaviour. This means that the semantics gives the same meaning to two commands if and only if they induce the same partial correctness behaviour in all program contexts. The semantics is thus well suited for compositional reasoning about program behaviour. The model uses sets of “transition traces”, closed under two natural conditions (“stuttering” and “mumbling”). The closed sets of traces, ordered by set inclusion, form a complete lattice; all functions on trace sets used in the semantic definitions are continuous. We show how to modify the semantics to handle different levels of granularity (e.g. interruptible assignment and expression evaluation), fair infinite computation, and deadlock. In each case we achieve full abstraction with respect to an appropriate notion of behaviour. Our treatment of fairness is based on Park’s “fairmerge” operator.

May I Borrow Your Logic?

MAURA CERIOLI and JOSÉ MESEGUER

It can be greatly advantageous to borrow key components of a logic for use in another logic. The advantages may be not only conceptual; due to the existence of software systems supporting mechanized reasoning in a given logic, it may be possible to reuse a system developed for one logic—for example, a theorem-prover—to obtain a new system for another. Translations between logics by appropriate mappings provide a first way of reusing tools of one logic in another. This work generalizes this idea to the case where entire components—for example, the proof theory—of one of the logics involved may be completely missing, so that the appropriate mapping could not even be defined. The idea then is to borrow the missing components (as well as their associated tools if they exist) from a logic that has them in order to create the full-fledged logic and tools that we desire. The relevant structure is transported using maps that only involve a limited aspect of the two logics in question—for example, their model theory. The constructions accomplishing this kind of borrowing of logical structure are very general and simple. They only depend upon a few abstract properties that hold under very general conditions given a pair of categories linked by adjoint functors.
The Category of Protocols

ROBIN COCKETT and DAVID SPOONER

There have been many models proposed for asynchronous systems. Most rely, following CCS, on an interleaved semantics of concurrency. We propose a new concrete model, which is truly concurrent, based on a special sort of transition system, which allows simultaneous actions from two independent agents. Such a transition system specifies the allowable interaction between agents, and is thus a communication protocol.

An original intention was to produce a natural model for a weakly distributive category with bang and whimper and to provide a semantics for the multiplicative and exponential fragment of linear logic (without negation). This nearly works except that the tensors are only associative if one admits bisimulation idempotents into the coherence diagrams. The interpretation of the tensor, cotensor (par), and exponentials we use is closely related to that employed in game theoretic models and there is a direct interpretation of this structure in process terms.

Of categorical interest is the manner in which model is developed: it is essentially a span category construction over a basic category of protocols quotiented by a "(strong) bisimulation" relation provided by a system of "open maps." Further, in order to isolate deterministic processes, we require that the two legs of the spans belong to two different classes of maps.

The spans of the base category turn out to provide inherently synchronous processes: asynchrony can be introduced through a Kleisli construction on the base category (which adds idle transitions). The Kleisli category which results has pullbacks (the monad is a commutative club) allowing the smooth formation of an appropriate span category at this level. Still unresolved, however, in the asynchronous case is the choice of the classes of maps which will yield an appropriate notion of bisimulation and determinism.

Finally, on a more pragmatic note: "well-designed" processes are often required to have further features, such as freedom from livelock and deadlock. These properties have corresponding categorical characterizations which are closed to the various constructions (composition, tensor, par, sum, parallel composition etc.) available in the setting: this promotes the modular design.
We investigate an operational model of concurrent systems, called automata with concurrency relations. These are labelled transition systems $A$ in which the event set is endowed with a collection of binary concurrency relations which indicate when two events, in a particular state of the automaton, can occur independently of each other. Similarly as in trace theory, through a permutation equivalence for computation sequences of $A$ we obtain an induced domain $(D(A), \leq)$.

In previous work, we studied the order structure of these domains and the relationship between these automata and Petri nets. Here, we construct a categorical equivalence between a large category of “cancellative” automata with concurrency relations and the associated domains. We show that each cancellative automaton can be reduced to a minimal cancellative automaton generating, up to isomorphism, the same domain. Furthermore, when fixing the event set, this minimal automaton is unique.

Hypercoherences

THOMAS EHRHARD

Some years ago, we observed that the Vuillemin-Milner definition of sequentiality could be expressed as a preservation condition similar to stability and we showed how this new approach allowed for a natural extension of “sequentiality” to higher order. This was a common work with Antonio Bucciarelli (cf. LICS '91 and ICALP '91).

Hypercoherences constitute a framework where all the constructions involved in this previous work can be expressed in a simpler way. We get this way a model of PCF (and various other kinds of $\lambda$-calculi) where types are interpreted by some kind of “reflexive and symmetric hypergraphs” (just as in coherence spaces, where types are interpreted by reflexive and symmetric binary graphs). Furthermore, these objects also provide us with a new model of “classical” linear logic.
Algebraically Compact Categories
MARCELO FIORE and GORDON D. PLOTKIN

Following Freyd a category is called algebraically compact if every endofunctor on it has a free algebra in the sense that it is an initial algebra whose inverse is a final coalgebra.

In this talk, the appropriateness of algebraically compact categories as a universe for interpreting recursive types (i.e. fixed-points of bifunctors) is addressed. First, a simple justification of the compactness axiom is proposed. Second, the fundamental property of algebraically compact categories (viz. that every bifunctor on an algebraically compact category universally induces an endofunctor whose free algebra provides a canonical and minimal fixed-point for the bifunctor) is established. Finally, in algebraically compact categories, recursive types are shown to be reducible to inductive types (i.e. fixed-points of endofunctors).

Lower Bag Domains
REINHOLD HECKMANN

In the classical power domain constructions, multiplicities of results are not taken into account. Together with the order structure, this implies that certain finite sets are identified. In contrast, we propose to count results, thus considering bags instead of sets. Since the lower construction is the simplest of the three classical power constructions, we investigate its non-idempotent analogue, the lower bag domain construction.

According to the general theory of power constructions, it can be introduced in two ways: as 'initial' construction $\mathcal{L}_i$, where $\mathcal{L}_i X$ is the free lower monoid over $X$ in DCPO, or as 'final' construction $\mathcal{L}_f$, where $\mathcal{L}_f X = \left(\left[X \to N^\infty_0\right] \xrightarrow{add} N^\infty_0\right)$ is the dcpo of continuous additive functions from $[X \to N^\infty_0]$ to $N^\infty_0$. The latter can be simplified to $\mathcal{L}_m X = \left[\Omega X \xrightarrow{mod} N^\infty_0\right]$, the dcpo of continuous modular functions from the lattice $\Omega X$ of opens of $X$ to $N^\infty_0$.

We show that for sober dcpo's $X$, each element of $\mathcal{L}_m X$ can be represented as a (possibly infinite) sum of singletons. For algebraic $X$, $\mathcal{L}_m X$ is again algebraic; its basis consists of finite sums of singletons from the basis of $X$. Equivalently, it consists of all finite bags of finite elements of $X$ ordered by an analogue of the lower order on sets. In contrast to the set case, all finite bags can be distinguished. From this description, one concludes that $\mathcal{L}_m X$ and $\mathcal{L}_i X$ are isomorphic for algebraic $X$ and hence for continuous $X$.

The construction $\mathcal{L}_i = \mathcal{L}_m$ does not preserve the property to be a Scott domain, nor to be bifinite. This negative result can be seen at a small finite example. Moreover, we can show that there is no power domain construction with characteristic semiring $N^\infty_0$ that preserves these domain classes.
A Theory of Classes: Proofs and Models

BARNEY P. HILKEN and DAVID E. RYDEHEARD

We investigate the proof structure and semantics of theories of classes, where classes are 'collections' of entities. The theories are weaker than set theories and arise from a study of type classes in programming languages, as well as comprehension schemata in categories.

We consider various languages of proofs for theories of classes and show how type classes arise through the use of proof environments. Normalisation results for proofs are obtained using long $\beta\eta$-normal forms. We use these results to define a semantics of proofs in terms of indexed categories with comprehension schemata.

Looking for Lambda Models

KARL H. HOFMANN and MICHAEL W. MISLOVE

This talk focuses on the question, "which categories contain a (nondegenerate) model of the untyped $\lambda$-calculus?" Using the notion of a functional domain [Meyer82]: "a set $D$ together with a set of selfmaps $[D \to D]$ which is a retract of $D$", we review known results. Namely, there are no nondegenerate models in SET, POSET or CU (complete ultrametric spaces and non-expansive maps). Then we present a new result: there are no compact Hausdorff models which are nondegenerate. The goal, of course, is to show there are no such models in the cartesian closed category $K$ of Hausdorff $k$-spaces and continuous maps. However, that result remains unsettled.

References


Elimination of extensionality for Martin-Löf Type Theory

MARTIN HOFMANN

Martin-Löf's intensional equality type is an attempt to apply the formulae-as-types paradigm to equality. Although very comfortable and probably the best one can expect, this "propositional" equality is incomplete since it lacks the principle of extensionality for function types. If the principle is simply assumed as an axiom then — because of the identification of propositions and types — noncanonical elements are introduced in all types, even for example in the type of natural numbers.
The problem can be solved by defining a syntactic transformation of proofs containing the extensionality axiom into proofs in the pure theory. This is done by constructing a categorical model out of the syntax in which the principle of extensionality is valid. The interpretation of the syntax in the model then provides the desired translation.

In the model types are interpreted as types equipped with equivalence relations and functions are interpreted as functions together with proofs that these relations are respected. In this sense the construction resembles the well-known elimination of extensionality for e.g. first-order logic. A new feature is the interpretation of dependent types which makes use of internal fibrations.

The elimination procedure can be extended to account for quotient types and coinductive types which also require extensional proof principles. These extensions are still work in progress.

Fair Games and Full Completeness for Multiplicative Linear Logic Without the MIX-Rule

J. MARTIN E. HYLAND and CHI-HAO LUKE ONG

We introduce a new category of finite, fair games and winning strategies, and use it to provide a semantics for the multiplicative fragment of Linear Logic (MLL) in which formulae are interpreted as games, and proofs as winning strategies. This interpretation provides a categorical model of MLL which satisfies the property that every (history-free, uniformly) winning strategy is the denotation of a unique cut-free proof net. Abramsky and Jagadeesan first proved a result of this kind and they refer to this property as full completeness. Our result differs from theirs in one important aspect: the MIX-rule, which is not part of Girard’s Linear Logic, is invalidated in our model. We achieve this sharper characterization by considering fair games. A finite, fair game is specified by the following data:

- moves which Player can play,
- moves which Opponent can play, and
- a collection of finite sequences of maximal (or terminal) positions of the game which are deemed to be fair.

Notably, positions of a game are a derived notion. The maximal positions of a compound game are obtained by an appropriate interleaving of the maximal positions of the respective constituent games. At any position in a finite, fair game, a player can make a move if, and only if, the move can be extended to a maximal position.
Full Intuitionistic Linear Logic

J. MARTIN E. HYLAND and VALERIA C.V. DE PAIVA

In this paper we give a brief treatment of a theory of proofs for a system of Full Intuitionistic Linear Logic. Since Girard and Lafont's original paper on Intuitionistic Linear Logic it seems to have been generally assumed that the multiplicative disjunction, \( \text{par} \), does not make sense outside the context of Classical Linear Logic; in particular \( \text{par} \) was thought to present problems for an interpretation of proofs as functions. However the connective \( \text{par} \) does have an entirely natural interpretation in models which appeared in the second author's doctoral dissertation, and it is our intention to make good the claim that full intuitionistic is a significant dialect of linear logic.

The novelty of Full Intuitionistic Linear Logic lies in its treatment of the interaction between \( \text{par} \) (which has the same 2-sided sequent-calculus rules as the Classical Linear Logic \( \text{par} \)) and the linear implication (which only satisfies the intuitionistic properties of an implication). We take as the main initial problem to be overcome the observation of Schellinx that cut elimination fails outright for the intuitive formulation of such a logical system. Our response is to develop a term assignment system which gives an interpretation of proofs as some kind of non-deterministic function (which appears as a sequence of partial functions evaluated in parallel). In this way we find a system which \textit{does} enjoy cut elimination. The system is a direct result of an analysis of the categorical semantics, though we make an effort to present the system as if it were purely a proof-theoretic construction.

Fibrations, Relational Databases and Schema Integration

AMITAVO ISLAM and WESLEY PHOA

Working within the powerdomain-theoretic framework of Buneman, Jung and Ohori, we give a categorical definition of (typed) relational schema (with functional constraints), and investigate how such schemata may be integrated.

A schema consists of a preorder \( \mathcal{L} \) of labels and abstract constraints, together with a typing functor \( V : \mathcal{L} \rightarrow (\text{Scott domains}) \). A record is a section of the corresponding fibration. Given a family of such schemata, and "glueing information" in the form of auxiliary schemata, we may combine them to obtain a federated schema. We show that queries on a database over a federated schema may be distributed among its components.
Parameters and Parametrization

BART JACOBS

We study some elementary matters in the semantics of datatypes. Specifically, models of (distributive) signatures will be described in terms of functors (after Lawvere) and models of parametrized signatures in terms of Kan extensions (after Reichel and Kaphengst). It is shown how models of the particularly simple subclass of Hagino signatures fit in; these are usually described in terms of algebras and coalgebras for an associated polynomial functor.

In a next step a completely general description of models with additional parameters is given in terms of what we call 'simple slice categories'. It specializes to the definition proposed by Cockett and Spencer for Hagino signatures.

The Semantics of Vectors

C. BARRY JAY

Not all list operations are vector operations. When computing a list, e.g. of solutions to an equation, its length may depend on the values appearing in the input list, not merely its length, and so may only be known at runtime. For vector computations, the length of the result is determined by the length (or, more generally, the shape) of the input. Often the length is unchanged (i.e. is an array operation), but it may easily increase or decrease by 1, or double, etc.

This distinction leads to a semantics of vectors and matrices (which are vectors of vectors) in which inner products of vectors and matrix multiplication can be described as total operations (instead of partial operations on pairs of lists and lists of lists, respectively). As an application, the discrete and fast Fourier transforms are modelled in this setting and shown to be equivalent.

A Cartesian Closed Category with Stable and Continuous Functions

FRANÇOIS LAMARCHE

We construct a category of “interpolative domains” which is cartesian closed and fills the gap between the world of stable functions à la Berry-Girard and the ordinary one of Scott-continuous functions.

An interpolative domain is a triple $(X, E, \leq)$ where $(X, E)$ is a prime-generated Scott domain and $\leq$ a suborder of $E$ satisfying certain conditions, that allows in particular the construction of a factorization system on $(X, E)$. If $\leq$ is the full order $E$ then $X$ acts as an ordinary Scott domain; if $\leq$ is identity $X$ acts as a stable domain. Intermediary choices for $\leq$ give rise to domains that are midways between the stable and the continuous world.
The syntactic calculus, a fragment of non-commutative linear logic, was introduced in 1958 because of its hoped for linguistic applications. Working with a Gentzen style presentation, one was led to the problem of finding all derivations \( f : A_1 \ldots A_n \rightarrow B \) in the free syntactic calculus generated by a context-free grammar \( G \) (with arrows reversed) and to the problem of determining all equations \( f = g \) between two such derivations. The first problem was solved by showing that \( f \) is equal to a derivation in normal form, whose construction involves no identities and no cuts (except those in \( G \)) and the second problem is solved by reducing both \( f \) and \( g \) to normal form.

Actually, the original motivation for the syntactic calculus came from multilinear algebra and a semantics was given by the calculus of bimodules. Bimodules \( R F S \) may be viewed as additive functors \( R \rightarrow \text{Mod} S \), where \( R \) and \( S \) are rings (of several objects). It is now clear that Lawvere’s generalized bimodules will also provide a semantics for what may be called typed bilinear logic.

Evaluation Logic
(Towards Integration with Synthetic Domain Theory)

EUGENIO MOGGI

This talk reports about some progress on the internal semantics of Evaluation Logic, which was first proposed at the 1991 Durham Meeting. This work originated by the inability of the original internal semantics to deal with powerdomains and monads for continuations. Our main result is that an internal semantics for the modalities of Evaluation Logic, which makes no additional assumptions on a strong monad.

Although it needs stronger assumptions about the ambient category and the class of monos used for interpreting logical formulas. These stronger assumptions are not satisfied by the category of cpos (and the class of its regular monos), but they hold in models for Synthetic Domain Theory, where the ambient category is usually a topos.

In fact, we have identified five ways of assigning meaning to necessity for a given strong monad (the new semantics is one of them). However, all of them agree with the original semantics of necessity, when this is defined.

As further research, we would like to give a sound (and complete) set of axioms for the new semantics of necessity, and to find (pragmatic) criteria for preferring one among the five possible semantics identified so far.
Categories and Computational Complexity

ADAM OBTULOWICZ

We introduce a new, machine independent hierarchy of conceptual (and geometrical) complexity of computable (labelled) graphs. The hierarchy is defined in terms of categories of diagrams and partial homomorphisms. The concepts of a graphical sketch and a higher level (an n-th level) graphical sketch are the basic new notions used in the definition of the considered hierarchy. The levels of graphical sketches correspond to the levels of the considered hierarchy.

The graphical sketches are diagrams valued in appropriate category of graphs and partial homomorphisms of graphs. The graphical sketches are the sketches of 1-st level. The n-th level graphical sketches are the diagrams valued in appropriate category of diagrams of level n−1. The graphical sketches and higher level graphical sketches present decompositions of infinite graphs into finite edgeswise disjoint subgraphs.

Parametricity and Local Variables, Part 1.5

PETER W. O’HEARN

This work in progress is a follow-on to P.W. O’Hearn and R.D. Tennent, “Relational Parametricity and Local Variables” from the 1993 POPL. A view of Algol types as polymorphic types is shown which underlies the relational-parametricity model. This makes clear the polymorphic nature of variable allocation: reasoning about programs in a language with local variables often reduces to proving properties of polymorphic functions. The polymorphism can be explained using Reynolds’s relational parametricity, as in O’Hearn-Tennent, or with PERs, or other reasonably parametric methods of interpreting polymorphism. In each case we get a model of Algol.

The polymorphic view immediately yields “compact” representations for 1st-order Algol types, derived from results connecting the denotations of certain polymorphic types to free algebras (Reynolds, Hyland, Freyd, Robinson, Rosolini). Some examples of reasoning about local variables using PERs are given. It is not known at present which of the PER and relational approaches is the best for Algol.
A Non-Well-Founded Approach to Subsumption in Knowledge Representation Languages

FRANK J. OLES
(joint work with ROBERT DIONNE and ERIC MAYS)

After discussing the basic constructs in knowledge representation (KR) languages, we turn to the problem of defining subsumption in such languages in the presence of recursive definitions. For a fragment of a typical KR language, K-REP, we give definitions of descriptive subsumption, structural subsumption and extensional subsumption. Descriptive subsumption, which is strictly weaker than the other two, is purely an algebraic concept. Structural subsumption is defined using Aczel’s theory of non-well-founded sets. Extensional subsumption is based on set-theoretic greatest fixed point models. The main result is that structural subsumption is equivalent to extensional subsumption for this fragment of K-REP.

What’s new?

ANDREW M. PITTS
(joint work with IAN STARK)

The research reported in this talk is concerned with the problem of reasoning about properties of higher order functions involving state. It is motivated by the desire to identify what, if any, are the difficulties created purely by locality of state, independent of other properties such as side-effects, exceptional termination and non-termination due to recursion. I consider a simple language (equivalent to a fragment of Standard ML) of typed, higher order functions that can dynamically create fresh names; names are created with local scope, can be tested for equality and can be passed around via function application, but that is all. Despite the extreme simplicity of the language and its operational semantics, the observable properties of such functions are shown to be very subtle. A notion of ‘logical relation’ is introduced which incorporates a version of representation independence for local names. I show how to use it to establish observational equivalences. The method is complete (and decidable) for expressions of first order types, but incomplete at higher types.
An Algebraic Approach to Object Semantics Based on Final Co-algebras

HORST REICHEL

Final co-algebras are mostly interpreted as an approach to deal with infinite data. We interpret a final co-algebra as a collection of all possible behaviours of sequential processes of a corresponding common type and therefore a final co-algebra may be used as a type for objects in the sense of object-oriented programming. In this way the representation type and the implementation of the methods for an object instance is hidden and may vary from one object instance to another one. It is illustrated that co-induction is the basic mechanism to create object instances, to define derived methods, and to deal with inheritance. Finally it is shown how this approach can be combined with J. Meseguer’s Rewriting Logic to specify the behaviour of concurrent object systems.

Naïve Synthetic Domain Theory — A Logical Account

BERNHARD REUS and THOMAS STREICHER

In our paper we show that Synthetic Domain Theory — as introduced by M. Hyland and P. Taylor on a category-theoretic basis — can be developed in purely logical terms using the language of higher order intuitionistic logic with subtype formation. Based on the few axioms proposed by P. Taylor in his LICS’91 paper we develop the theory of complete extensional pers and the theory of replete objects and show that they provide an appropriate notion of predomain (where for the first case we need Markov’s principle). Using the diagramatic language only for the purpose of illustration we prove in full detail that

- all functions between predomains are continuous
- any endofunction between predomains with a least element admits a least fixpoint
- the fixpoint induction principle holds

Finally we sketch the proof that any internal endofunctor on the category of domains and strict maps admits an initial fixpoint.

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Normalization for Lambda Calculi with Explicit Definitions

EIKE RITTER

The strong normalization for lambda calculi with explicit definitions is a longstanding open problem. I will first explain why the standard approach for showing strong normalization in the simply-typed lambda calculus does not work in this case. Then I will show that any notion of reduction that reduces a term first to weak head-normal form and only then pushes the evaluation of a definition under the lambda-abstraction is strongly normalizing. This includes all practically relevant reduction strategies like lazy and eager evaluation. Hence the problem of designing efficient implementations for the evaluation of definitions has been separated from showing that the underlying reduction strategy always terminates.

Parametricity and Data Abstraction

EDMUND P. ROBINSON

The notion of a “parametrically” polymorphic operation was introduced originally by Strachey in order to refer to operations which could be applied at different types, but for which the code at the different types was identical. Subsequently, John Reynolds observed that there was a connection between this notion, the concept of data abstraction, and the tool of logical relations. We begin by giving an example of how this connection is supposed to function, and surveying work in the field of relational parametricity. We then firm up our ideas by looking at different formulations and levels of data abstraction, showing that for a simple specification-oriented form of abstraction which allows copying of pre-existing types there is a connection with relational parametricity in which the relations are the graphs of isomorphisms. The talk ends with a proposal for capturing a more model-oriented form of data abstraction.

On a construction of Carboni’s

GIUSEPPE ROSOLINI

We know that the effective topos $\mathcal{E}$ is determined by an exact completion of a particular category, and A. Carboni has improved this characterization showing that the topos satisfies a universal property with respect to the inclusion $R \hookrightarrow \mathcal{S}$ of the category of subsets of $N$ and partial recursive functions, see [1].

The interest of the construction lies in two facts: the effective topos is one of the first workbenches where any algebraic notion about computations gets tested, and the exact completion of a category has a very syntactic nature. For instance, if a left exact category has disjoint stable coproducts, or is (locally) cartesian closed, or has a natural number object, then so does respectively its exact completion.

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We show that the exact completion of the internal presentation of $R$ in $\text{Eff}$ represents the fibration of discrete objects over all separated objects, see [2]. We prove also that $R_{\text{ex}}$ is locally cartesian closed by direct computation, and that it may sustain a model of polymorphism. It is appropriate to make the trivial remark that the model would be exact, therefore allowing a nice factorization system for the arrows.

References


Weakly Distributive Categories —
Proof Theory, Coherence, and Tensorial Strength

ROBERT A.G. SEELY
(joint work with RICHARD BLUTE, ROBIN COCKETT, T. TRIMBLE)

The notion of weakly distributive categories was developed (partly) in order to study the situation in which one has two “dual” tensor structures (e.g. tensor and par in linear logic), without having the duality explicit (i.e. without having a negation present). The minimal “reasonable” such notion would seem to involve having each tensor possess a “strength” with respect to the other (the “weak distributivity” laws). We develop the proof theory of weakly distributive categories in terms of proof nets, and derive a coherence theorem; in order to accommodate the thinning introduced by the units, we need to add a new kind of link (“thinning links”) to the traditional proof nets. As a corollary, we show that the extension from weakly distributive categories to $\ast$-autonomous categories is strongly conservative (in the sense that the unit of the adjunction is fully faithful).

We can use the trick of thinning links (together with the more traditional “storage boxes”) to extend these nets to include the storage modalities “bang” ($!$) and “whimper” ($?$). We again find that a crucial notion in formulating the appropriate structure of weakly distributive categories with storage (to capture the implicit duality between $!$ and $?$ without having an explicit negation) is a generalization of tensorial strength: each functor ($!$, $?$) must be strong with respect to the appropriate tensor and relative to the other. In this context it is possible to obtain a coherence theorem similar to that for weakly distributive categories. (This result is still work-in-progress.)
New Steps Towards Full Abstraction for Local Variables

KURT SIEBER

We present a denotational semantics for an ALGOL-like language ALG. ALG has—as usual for ALGOL-like languages—block structure, local variables, arbitrary higher types and—as a particular ingredient—expressions with temporary side effects (‘snap back semantics’).

It is well known that traditional denotational models for ALGOL-like languages with local variables are not fully abstract, i.e. they fail to prove some (important) observational congruences. Research in the last ten years led to improved models, but even for the most recent ones a proof of full abstraction is still missing.

Our new model is based on ‘relationally structured locally complete partial orders’ and ‘relation preserving locally continuous functions’ as introduced in [MS88], but it has a finer ‘relation structure’ than the model in [MS88]. Like the model in [OT93] it can be used to prove all (problematic) observational congruences which have been presented in the literature. Moreover, our model is ‘small’ in the sense that for every function \( f \) of order \( \leq 2 \) there is a definable function which coincides with \( f \) on finitely many given arguments. We believe that this result is a promising (and necessary) step towards proving full abstraction for the second order sublanguage of ALG.

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Stone Duality in Synthetic Domain Theory

PAUL TAYLOR

Synthetic domain theory has been guided so far by the slogan that domains and sets and all functions are continuous. Taken literally, this just says that the category of (pre)domains is to be a full subcategory of a (necessarily non-classical) category of sets, i.e. a topos, but the ease with which the Yoneda embedding achieves this means that there can be no profit from this interpretation. Wesley Phoa’s thesis strengthened this to a reflective subcategory closed under exponentials, which was selected from a given world by a Leibniz principle on points together with considerations of continuity. My own paper in LiCS ’91 introduced a Leibniz principle on objects and showed that the adhoc continuity condition on each object could be transferred to one on the universe.

The present work is motivated by the feeling that an even stronger meaning can be given to the slogan: how many of the features of sets can we demand of predomains, given that they must also satisfy fixed point properties?

As I had previously been working on the presentation of regular categories, pretoposes and while programs for my book Practical Foundations, I began investigating this question by postulating several inter-related properties. Although not on the original list, the following is typical of them and has emerged as the crucial one:

If \( m : X \to Y \) is a regular mono in the category of replete objects (predomains) then \( \Sigma^m : \Sigma^Y \to \Sigma^X \) is stably regular epi.

Essentially the same question arose from Eugenio Moggi’s work in Evaluation Logic, where predicates are to be interpreted using monos from a class which must be preserved by the functors of interest. He gave a counterexample to show that this property fails for any category of domains where the order determines the topology, so to find models we must go outside the familiar framework: to general topological spaces, locales, filter spaces or something else.

Besides the finitary exactness properties which are captured in the notion of pretopos, the category \( \mathcal{C} \) of sets exhibits the following phenomenon, in which \( \Sigma \) denotes the object of truth values, more usually called \( \Omega \):

- \( \mathcal{C} \) has exponentials of the form \( \Sigma^X \) (powersets) and the functor \( \Sigma(-) : \mathcal{C} \to \mathcal{C}^{\text{op}} \) is symmetrically adjoint to itself on the right. Moreover this adjunction is monadic. That is, \( \mathcal{C} \) is dual to a category of algebras over itself.

This result, due to Bob Paré and dating from the early days of elementary topos theory, is a pure form of Stone Duality. From another point of view we may see it as a Leibniz principle on categories: whereas the earlier principles identified points sharing the same properties and domains with the same lattice of properties, this identifies categories which have the same category of property-lattices. \( \text{Alg} \) captures what it is to be a category of such lattices, because of the following result:
Let \( C \) be any category with finite products, splittings of retracts, and exponentials \( \Sigma^X \) of a pointed object \( \Sigma \). Let \( \text{Alg} \) be the category of Eilenberg-Moore algebras for the monad \( \Sigma^{E^\to} \) arising from the adjunction. Then \( \text{C} = \text{Alg}^{\text{op}} \) has the same properties as \( C \) (with \( \Sigma = \Sigma^{E} \) playing the rôle of \( \Sigma \)), except that now the adjunction is monadic. The inclusion \( C \to \text{C} \) preserves finite products and exponentials of \( \Sigma \).

In other words, although toposes are the only hitherto known examples of Paré's result, the situation is easy to create.

It is well known that limits of algebras are calculated as for the underlying objects, but colimits of algebras — other than the peculiar "reflexive \( U \)-contractible" ones arising from John Beck's characterisation of monadic adjunctions — are notoriously difficult to find. It took me two months to show that \( \text{C} \) has binary products to give meaning to the above result, and I have very little information about the equalizers and pullbacks.

Indeed the above question about regular monos and epis seems to be exactly what we must ask of the concrete examples (dcpo's, pers) to make progress here.

The Paré property for \( \text{C} \) does however pay rich dividends:

- \( \text{C} \) is cartesian closed;
- \( \Sigma \) is a dominance if \( \Sigma \) has a \( \Lambda \)-semilattice structure, i.e. the inverse image of \( T \to \Sigma \) along \( f : X \to \Sigma \) determines \( f \), and the class of monos arising in this way is closed under pullback;
- \( \text{C} \) is cocomplete if \( C \) is complete;
- if \( \text{C} \) is absolutely cocomplete (as defined by Ross Street) then it is the category of replete objects with respect to \( \Sigma \) within the topos of sheaves for the canonical topology on \( \text{C} \).

So although I have been unable to construct general equalisers in \( \text{C} \) when \( C \) is a category of spaces, the above results invite application to the small complete categories which are known to arise in realisability examples.

With reference to the classical (topological) setting, it appears that known categories such as those of filter spaces and locally compact locales already lie within \( \text{C} \) with just \( C = \text{Dcpo} \) as underlying objects. This means that \( \text{C} \) cannot be generalised and so is canonical in a way which other categories of spaces are not. Also, instead of jettisoning rafts of inconvenient objects, cartesian closure is achieved by enlarging the category of spaces.

Now that the full higher-order logic of toposes and the \( \Sigma^0 \) logic of computation have been presented in a common way, one might suppose that other fragments may also be treated. I am in particular interested in one which is closed under universal quantification, with a view to relativising the notion of well-foundedness.

In conclusion, this piece of work has proved to be more difficult than was originally imagined, though this is in line with the boldness of the proposal. However what has been achieved suggests that some very powerful results may be obtained by a future application to the realisability examples.
Towards Single-Threaded Semantics of Algol-like Languages

ROBERT D. TENNENT

The conventional semantic framework for imperative languages allows linguistic features (such as \textsc{RESET}, to re-initialize the entire state) which are very inefficient and are not encountered in practical languages. The aim of the work discussed in this talk is to find a framework for "single-threaded" semantics, in which such constructs cannot be defined. The motivations include

- a better match between theory and practice
- full abstraction and
- simplification of the task of semantics-directed compilation.

The first idea that occurs is that single-threadedness is captured by linearity; i.e. it should not be possible to duplicate or discard a state. But this is too naive. For example, the conventional semantics of expression forms discards states and specifies that they be duplicated.

Rather than abandon the concept of linearity, we propose an alternative treatment of expressions: the basic idea is that expression evaluation propagates the state (unchanged) as well as yielding a value. This can be made to work with variable declarations and even block expressions by using a form of possible-world semantics that allows both state expansions and restrictions to singleton sets of states. The restrictions ensure that expression evaluations cannot have side effects to non-local variables, not even temporary ones. The resulting semantics integrates the stack of local variables, as in the Reynolds-Oles model, with the stack of temporary values produced during expression evaluation. Of course, this is exactly what happens in stack-machine implementations.
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