

# Battery Lifetime Distributions

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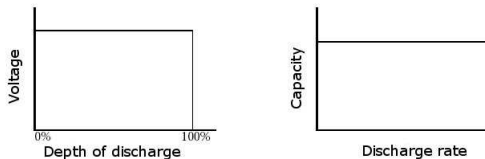
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# Battery lifetime

- essential for wireless devices
  - WLAN
  - GSM
  - sensor networks
- battery lifetime depends on
  - battery capacity
  - overall power consumption
  - low-level usage pattern
- usage pattern = workload  $\Rightarrow$  stochastic models

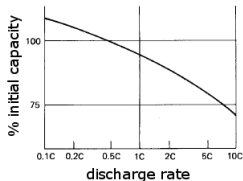
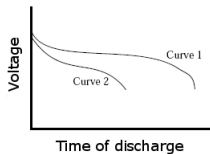
## Ideal batteries



- voltage ( $V$ ) constant during discharge
- capacity ( $C$ ) constant for the load current ( $I$ )
- battery lifetime ( $L$ ) is given by

$$L = \frac{C}{I}$$

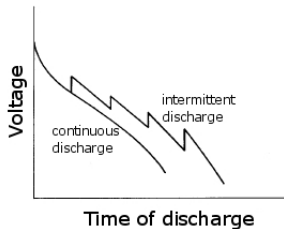
## Rate Capacity effect



- continuous drawing of a high current results in an excessive drop of residual capacity
- Peukert's Law for constant loads

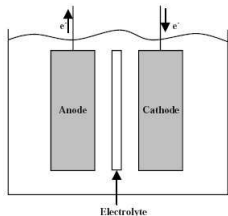
$$L = \frac{a}{I^b}$$

## Recovery effect



- during intervals with no or very small currents

## Nonlinear effects – physical explanation



- ions have to diffuse from anode to cathode
- at high currents the internal diffusion is too slow and
- no reaction sites (molecules) are available in the cathode
- during idle periods the ions have time enough to diffuse again

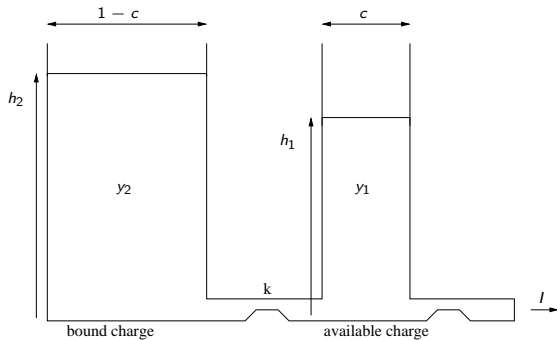
## Models on different levels

- electro-chemical models (Dualfoil)
- electrical circuit models (SPICE)
- analytical models (Peukert's Law, KiBaM)
- stochastic models

# Kinetic Battery Model (KiBaM)

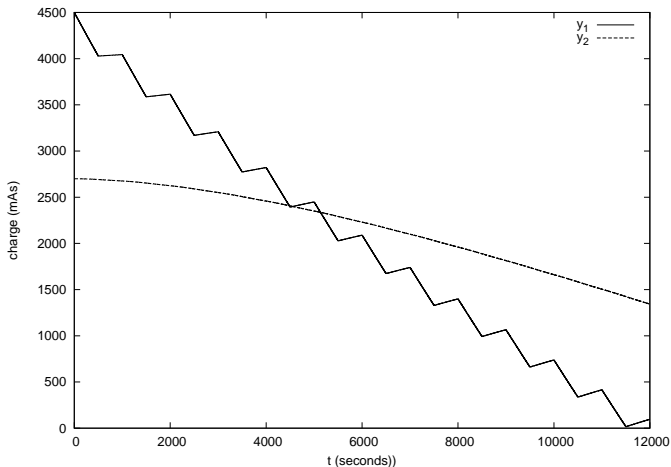
- battery charge divided in two wells
  - available-charge well
  - bound-charge well
- available-charge well supplies electrons to load
- bound-charge well supplies electrons to available-charge well
- differential equations describe evolution

# KiBaM – differential equations



$$\begin{aligned}\frac{dy_1}{dt} &= -I + k(h_2 - h_1), \\ \frac{dy_2}{dt} &= -k(h_2 - h_1),\end{aligned}$$

## KiBaM – discharge curves

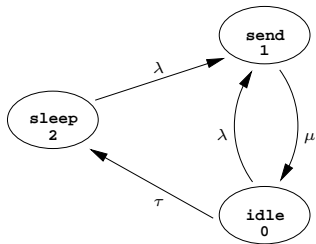


$$C = 7200As, I = 0.96A, k = 4.5 \cdot 10^{-5}, c = 0.625, \\ f = 0.001Hz$$

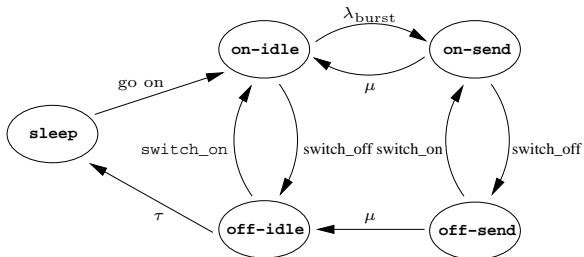
# KiBaM – evaluation

- reflects
  - Rate Capacity effect
  - Recovery effect
- experimental settings show that there is also frequency dependency
  - not exhibited in KiBaM

## Simple & burst model of a wireless device



- simple and burst model have the same steady-state probability to be in a send state
- $I_{\text{idle}} = 7\text{mA}$ ,  $I_{\text{send}} = 200\text{mA}$ ,  $I_{\text{sleep}} = 0\text{mA}$



## KiBaMRM

- two accumulated rewards
  - $Y_1(t)$ : available-charge well
  - $Y_2(t)$ : bound-charge well
- power consumption rate  $I_i$  in each state
- reward rates according to differential equations

$$r_{i,1}(y_1, y_2) = \begin{cases} -I_i + k \cdot (h_2 - h_1), & h_2 > h_1 > 0, \\ 0, & \text{otherwise,} \end{cases}$$

$$r_{i,2}(y_1, y_2) = \begin{cases} -k \cdot (h_2 - h_1), & h_2 > h_1 > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where  $h_1 = y_1/c$  and  $h_2 = y_2/(1 - c)$ .

- battery lifetime  $L = \min\{t \mid Y_1(t) = 0\}$  is a RV
- we compute the lifetime *distribution*

$$\Pr\{\text{battery empty at time } t\} = \Pr\{Y_1(t) = 0\}$$

# Inhomogeneous MRMs as stochastic (work)load models

- homogeneous case
  - state space  $S$  + generator matrix  $\mathbf{Q}$   
 $\Rightarrow$  stochastic process  $X(t)$
  - reward rates  $r_i$  for state  $i \in S$
  - accumulated reward

$$Y(t) = \int_0^t r_{X(s)} ds$$

- inhomogeneous case
  - $\mathbf{Q}(t, y)$  and  $r_i(t, y)$  depend on the current time  $t$  and the current accumulated reward

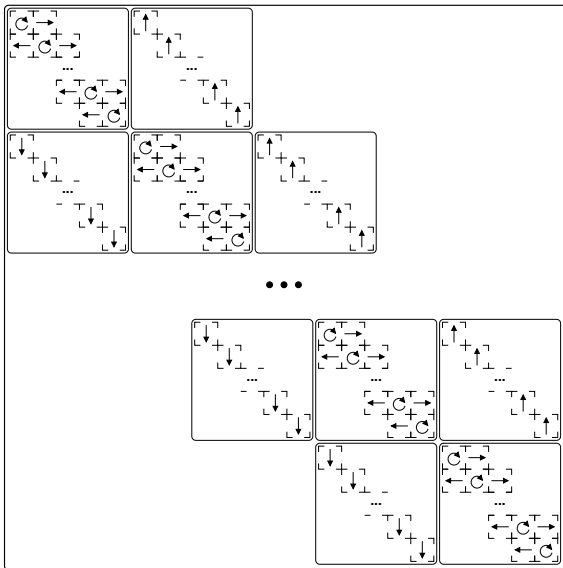
$$Y(t) = \int_0^t r_{X(s)}(s, Y(s)) ds$$

- battery case
  - more than one accumulated reward
  - reward rates depend on accumulated rewards, but not on time

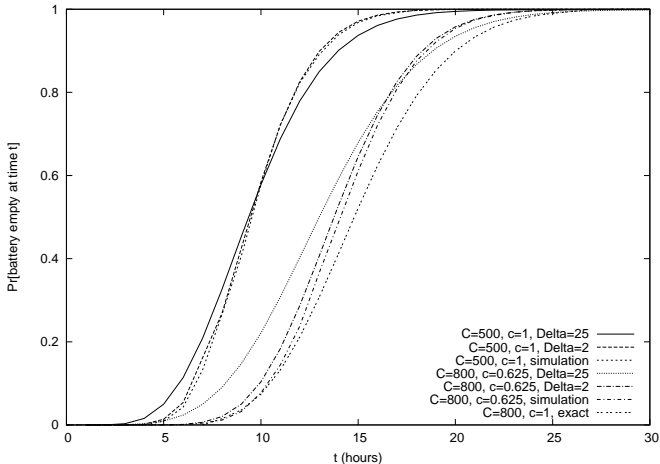
# Computation of the lifetime distribution

- needed: algorithm for transient distribution of accumulated reward in an reward-inhomogeneous MRM
- our choice: “Markovian Approximation”
  - discretise both accumulated rewards
  - CTMC with states  $(s, y_1, y_2)$
  - integral is replaced by sum over transient probabilities (computed with with uniformisation)
  - accuracy depends on the chosen stepsize  $\Delta$
  - no error bound known

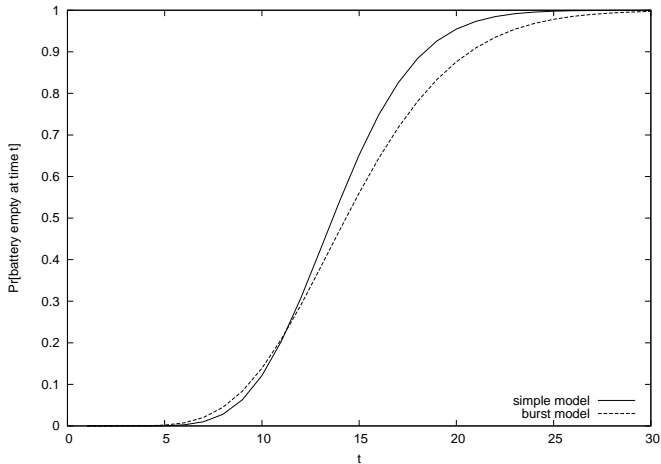
# Generator matrix



## Simple model - lifetime distribution



## Simple vs. burst model



$$C = 800\text{mAh}, c = 0.625$$

## Conclusion

- consider nonlinear aspects of battery lifetime
- combine with stochastic workload models
- KiBaM  $\implies$  KiBaMRM not yet sufficient
- MRMs are powerful enough to cover all aspects
- efficient algorithms