Schedule and Book of abstracts

Applied and Combinatorial Topology

Edited by

Paweł Dłotko, Dmitry Feichtner-Kozlov, Anastasios Stefanou, and Yusu Wang

All talks will take place in room **N 034 -** ***Lecture hall 2 - Karlsruhe***

Time	Monday	Tuesday	Wednesday	Thursday	Friday
7:30-8:45	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
9 AM-10 AM	**Organizers & Participants Introduce Themselves**	**Group (I)-Part A** (P. Dlotko) Topological descriptors for efficient analysis of electronic structures - Ingrid Hotz *Bounding the Interleaving Distance for Mapper Graphs with a Loss Function* -Elizabeth Munch *From Coarse to Fine and Back Again: Geometry and Topology in Machine Learning* -Bastian Rieck	**Group (I)-Part B** (Y. Wang) Barcodes for the topological analysis of gradient-like vector fields -Claudia Landi *Topologically Attributed Graphs* -Thomas Needham *(Discrete) Morse Theory and Reconstruction* -Julian Brüggemann	**Group (IV)** (D. Feichtner-Kozlov) *Directed paths and duality* -Martin Raussen *Large Simple d-Cycles in Simplicial Complexes* -Roy Meshulam **Group (III)-Part B** (A. Stefanou) *Hypergraph Barcodes: a way to Link two Different Notions of Hypergraph Homology* -Robert Green	**Brain-storming Session** **Groups (I) & (II)**
10 AM-11 AM	Coffee & Tea	Coffee & Tea	Coffee & Tea	Coffee & Tea	Coffee & Tea
11 AM-12 PM	**Introductory lectures** *Overview of Discrete Morse Theory Part I* -Leonard Wienke *Overview of Discrete Morse Theory Part II* -Nicholas Scoville	**Group (II)** (A. Stefanou) *When Do Two Distributions Yield the Same Expected Euler Characteristic Curve in the Thermodynamic Limit* -Niklas Hellmer *Magnitude, Alpha Magnitude and Applications* -Sara Kališnik Hintz *Combinatorial Topological Models for Phylogenetic Networks* -Jan Senge	**Group (III) -Part A** (A. Stefanou) *Multiparameter persistence is practical* -Michael Kerber *Models of Subdivision Bifiltrations* -Michael Lesnick *The discriminating power of the generalized rank invariant* -Woojin Kim	Merge Tree for Periodic Data -Teresa Heiss Challenges in two- and multi-parameter persistent cohomology -Fabian Lenzen *Persistent cup modules* -Ling Zhou	**Brain-storming Session** **Groups (III) & (IV)**
12 PM-1 PM	Lunch	Lunch	Lunch	Lunch	Lunch
1 PM-3 PM	*Discrete Morse theory and persistent homology of geometric complexes* -Ulrich Bauer *Computational Topology for Zigzag Persistence* -Tamal Dey *Reeb Graphs and Their Variants: Theory and Applications* -Bei Wang *A Statistical Perspective on Multiparameter Persistent Homology* -Mathieu Carrière	**2 Parallel Breakout Sessions:** **Group (I)** Mappers, Discrete Morse Theory & Statistics in TDA -Organizers: Y. Wang, P. Dlotko **Group (II)** New Invariants in TDA -Organizer: A. Stefanou	**Excursion** Trip to nearby Town with a Guided Tour (Cost: will be announced) *Boarding Bus at 13:15 PM *Arrive to Dagstuhl at 6 PM	**2 Parallel Breakout Sessions:** **Group (III)** Persistence in TDA -Organizer: A. Stefanou **Group (IV)** General Applied Topology -Organizer: D. Feichtner-Kozlov	End of Seminar
3 PM-4 PM	Coffee & Tea	Coffee & Tea	Coffee & Tea	Coffee & Tea	
4 PM-6 PM	**Open Problem Session** For the Groups (I), (II), (III), (IV)	**Breakout Sessions (Continued)**		**Breakout Sessions (Continued)**	
6 PM-7 PM	Dinner	Dinner	Dinner	Dinner	

Dagstuhl Reports, pp. 1–10

Dagstuhl Reports
Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

1 Table of Contents

Overview of Talks

2 Overview of Talks

2.1 Discrete Morse theory and persistent homology of geometric complexes

Ulrich Bauer (TU München, DE)

I will discuss the interplay between geometry and topology, and between Morse theory and persistent homology, in the setting of geometric complexes. This concerns constructions like Rips, Čech, Delaunay, and Wrap complexes, which are fundamental construction in topological data analysis. The tandem of Morse theory and homology shows the topological equivalence of several of these constructions, helps in speeding up their computation by a huge factor (in the software Ripser), reveals thresholds at which homology necessarily vanishes (with links to a classical result by Rips and Gromov), and relates optimal representative cycles for persistent homology to the industry-tested Wrap reconstruction algorithm.

2.2 (Discrete) Morse Theory and Reconstruction

Julian Brüggemann (Universität Bonn, DE)

Morse theory and its discrete version are well established toolboxes in pure topology. They both serve a similar purpose: use the combinatorics of the real numbers via well-behaved real-valued functions to compute topological invariants of geometric objects. In some instances, certain collections of topological invariants allow for a complete classification of the given class of spaces, which in turn might allow for a reconstruction of the original objects from the computed collection of invariants, most time up to some suitable notion of equivalence. In this talk, I will give a brief overview over smooth and discrete Morse theory and mention some classification/reconstruction results in topology and visualization that might also be useful in data analysis.

2.3 A Statistical Perspective on Multiparameter Persistent Homology

Mathieu Carrière (Centre Inria d'Université Côte d'Azur - Sophia Antipolis, FR)

Multiparameter persistent homology is a generalization of persistent homology that allows for more than a single filtration function. Such constructions arise naturally when considering data with outliers or variations in density, time-varying data, or functional data. Even though its algebraic roots are substantially more complicated, several new invariants have been proposed recently. In this talk, I will go over such invariants, as well as their stability, vectorizations and implementations in statistical machine learning.

2.4 Computational Topology for Zigzag Persistence

Tamal K. Dey (Purdue University - West Lafayette, US)

In topological data analysis, zigzag persistence has become an important component because it enhances the applicability of persistence theory by allowing both insertions and deletions of simplices in a simplicial filtration. Such filtrations occur in applications where a space or a function on it changes over time. For example, in network analysis, new connections appear and existing connections disappear over time. The standard persistence algorithm for non-zigzag filtrations does not work for the zigzag case. After laying out the background and earlier work on computations of zigzag persistence, we present a new algorithm FastZigzag for computing zigzag persistence from an input filtration. We follow it with the discussion of the well known vineyard problem in the zigzag case. We present a recent efficient algorithm for computing the zigzag vineyard. Akin to the non-zigzag case, the special but important case of graphs allow certain optimizations that make the computations of zigzag barcode and their vineyards more efficient. We go over some of these developments. Finally, we indicate some of the applications of zigzag persistence, in particular to data analysis in TDA with multiparameter persistence.

2.5 Hypergraph Barcodes: a way to Link two Different Notions of Hypergraph Homology

Robert Green (University at Albany, US)

Hypergraphs are a natural data structure to consider when studying networks with multiway connections. One approach to characterizing the features of these networks involves defining a form of hypergraph homology and then leveraging these homological traits to delineate the hypergraphs. There are many different ways however to define hypergraph homology and different approaches yield different types of features. In this talk I will present two different approaches to this problem and then connect them by presenting a persistence module they both live inside of.

2.6 Merge Tree for Periodic Data

Teresa Heiss (Institute of Science and Technology Austria, AT)

Periodic data is abundant in material science, for example the atoms of a crystalline material repeat periodically. Additionally, periodic boundary conditions are used in many further applications, for example in cosmology, to remove boundary effects. It is unclear how to deal with the periodicity of the data when computing topological descriptors, like the merge tree or persistent homology. A classical approach is to compute the respective descriptor simply on the torus. However, this does not give the information needed for many applications and is in some sense even unstable under noise. Therefore, we suggest decorating the periodic merge tree gained from the torus with additional information, describing for each connected component how many components of the infinite periodic covering space map to it. The resulting periodic merge tree carries the desired information and fulfills all the desired properties, in particular: stability and efficient computability.

2.7 When Do Two Distributions Yield the Same Expected Euler Characteristic Curve in the Thermodynamic Limit

Niklas Hellmer (Polish Academy of Science, PL)

Joint work of Hellmer, Niklas; Fleckenstein, Tobias
Main reference Hellmer, Niklas; Fleckenstein, Tobias, When Do Two Distributions Yield the Same Expected Euler Characteristic Curve in the Thermodynamic Limit?, ArXiv, 2024.
URL https://arxiv.org/abs/2401.04580

Given a probability distribution F on $\mathbb{R}^d$ with density f, consider a sample X_n of n points sampled from F i.i.d.. We study the Euler characteristic curve (ECC) of the union of balls $\bigcup_{x \in X_n} \overline{B}_{r_n}(x)$ in the thermodynamic limit. That is, as $n \to \infty$, we let $r_n \to 0$ such that nr_n^d approaches a finite, non-zero limit. It turns out that two distributions yield the same expected ECC in this setting if and only if they have the same excess mass. Whether this condition is also necessary for the distributions of the ECCs to coincide in the limit remains an open question.

2.8 Magnitude, Alpha Magnitude and Applications

Sara Kalisnik (ETH Zürich – Zürich, CH)

Joint work of Kalisnik, Sara; O'Malley, Miguel; Otter, Nina

Magnitude is an isometric invariant for metric spaces that was introduced by Leinster around 2010, and is currently the object of intense research, since it has been shown to encode many known invariants of metric spaces. In recent work, Govc and Hepworth introduced persistent magnitude, a numerical invariant of a filtered simplicial complex associated to a metric space. Inspired by Govc and Hepworth's definition, we introduced alpha magnitude. Alpha magnitude presents computational advantages over both magnitude as well as Rips magnitude, and is thus an easily computable new measure for the estimation of fractal dimensions of real-world data sets.

2.9 Multi-parameter persistence is practical

Michael Kerber (TU Graz, AT)

I will present some recent advances from our group at TU Graz that allow us to handle much larger bifiltered data sets with a computational pipeline than what was possible before.

2.10 The discriminating power of the generalized rank invariant

Woojin Kim (Duke University - Durham, US & KAIST - Daejeon, KR)

Joint work of Kim, Woojin; Clause, Nathaniel; Mémoli, Facundo
Main reference Clause,Nate; Kim, Woojin; Memoli, Facundo, The discriminating power of the generalized rank invariant, arXiv, 2023
URL https://arxiv.org/abs/2207.11591

In topological data analysis, the rank invariant is one of the best known invariants of persistence modules over posets. The rank invariant of a persistence module M over a given poset P is defined as the map that sends each comparable pair $p \leq q$ in P to the rank of the linear map $M(p \leq q)$. The recently introduced notion of generalized rank invariant acquires more discriminating power than the rank invariant at the expense of enlarging the domain of rank invariant to a collection I of intervals of P that contains all segments of P. In this talk, we discuss the tension that exists between computational efficiency and the discriminating power of the generalized rank invariant, depending on its domain I. The Möbius inversion formula will assume a significant role in clarifying the discriminating power, even in cases where the domain I is not locally finite. Along the way, we show that the possibility of encoding the generalized rank invariant of M over a non-locally-finite I into a multiset of signed intervals of P depends on how "tame" M is. Such a multiset, if it exists, is obtained via Möbius inversion of the generalized rank invariant over a suitable locally finite subset of I.

2.11 Barcodes for the topological analysis of gradient-like vector fields

Claudia Landi (University of Modena and Reggio Emilia, IT)

Joint work of Bannwart, Clemens; Landi, Claudia
Main reference Bannwart, Clemens; Landi Claudia, Barcodes for the topological analysis of gradient-like vector fields, arXiv, 2024
URL https://arxiv.org/abs/2401.08466

Intending to introduce a method for the topological analysis of fields, we present a pipeline that takes as an input a weighted and based chain complex, produces a tame epimorphic parametrized chain complex, and encodes it as a barcode of tagged intervals. We show how to apply this pipeline to the weighted and based chain complex of a gradient-like Morse-Smale vector field on a compact Riemannian manifold in both the smooth and discrete settings. Interestingly for computations, it turns out that there is an isometry between tame epimorphic parametrized chain complexes endowed with the interleaving distance and barcodes of tagged intervals endowed with the bottleneck distance. Concerning stability, we show that the map taking a generic enough gradient-like vector field to its barcode of tagged intervals is continuous. Finally, we prove that the barcode of any such vector field can be approximated by the barcode of a combinatorial version of it with arbitrary precision.

2.12 Challenges in two- and multi-parameter persistent cohomology

Fabian Lenzen (TU Berlin, DE)

In the last years, research in persistent homology has started to focus on multi-parameter persistent homology, which studies the homology of a space filtered by multiple parameters independently. For example, this can be used to overcome the notorious susceptibility of persistent homology to outliers, to deal with data sats of inhomogeneous density, or to study filtration types that rely on more than one parameter.

Computing multi-parameter persistent homology is challenging, both algebraically and algorithmically. In particular, current software is orders of magnitudes slower than common software for one-parameter persistence.

We will discover why persistent cohomology—a key ingredient in the efficiency of one-parameter persistence software—is inherently more difficult in multi-parameter persistence, how this is dealt with in the software package 2pac, and what problems still remain.

2.13 Models of Subdivision Bifiltrations

Michael Lesnick (University at Albany – New York, US)

Joint work of Lesnick, Michael; McCabe, Kevin

We study the size of Sheehy's subdivision bifiltrations, up to homotopy. We focus in particular on the subdivision-Rips bifiltration $\mathcal{SR}$, the only density-sensitive bifiltration on metric spaces known to satisfy a strong robustness property. Given a simplicial filtration $\mathcal{F}$ with a total of m maximal simplices across all indices, we introduce a simplicial model for its subdivision bifiltration $\mathcal{SF}$ whose k-skeleton has size $O(m^{k+1})$. We also show that the 0-skeleton of any simplicial model of $\mathcal{SF}$ has size at least m. We give several applications: For arbitrary metric spaces, we introduce a $\sqrt{2}$-approximation to $\mathcal{SF}$ with poly-size skeleta, improving on the previous best approximation bound of $\sqrt{3}$. Moreover, we show that the approximation factor of $\sqrt{2}$ is tight; in particular, there exists no exact model of $\mathcal{SR}$ with poly-size skeleta. On the other hand, we show that for data in a fixed-dimensional Euclidean space with the ℓ_p-metric, there exists an exact model of $\mathcal{SR}$ with poly-size skeleta for $p \in \{1, \infty\}$, as well as a $(1+\epsilon)$-approximation to $\mathcal{SR}$ with poly-size skeleta for any $p \in (1, \infty)$ and fixed $\epsilon > 0$.

2.14 Large Simple d-Cycles in Simplicial Complexes

Roy Meshulam (Technion - Haifa, IL)

Joint work of Meshulam, Roy; Newman, Ilan; Rabinovich, Yuri

Let $G = (V, E)$ be a finite simple graph. A classical result of Erdos and Gallai asserts that if $|E| > \frac{k(|V|-1)}{2}$, then G contains a simple cycle of length $> k$. We study the analogous question for higher dimensional simplicial complexes. A set $\{\sigma_1, \ldots, \sigma_k\}$ of d-dimensional simplices in a simplicial complex X is a *simple d-cycle over a field F* if $\{\partial\sigma_1, \ldots, \partial\sigma_k\}$ is a minimal linearly dependent set in the space of d-chains $C_d(X; F)$. Let $f_i(X)$ denote the number of i-dimensional simplices in X. It is shown that any d-dimensional X contains a simple d-cycle of size

$$k \geq \sqrt{\frac{2f_d(X)}{(d+1)f_{d-1}(X)}} - 1.$$

2.15 Bounding the Interleaving Distance for Mapper Graphs with a Loss Function

Elizabeth Munch (Michigan State University, US)

Data consisting of a graph with a function to $\mathbb{R}^d$ arise in many data applications, encompassing structures such as Reeb graphs, geometric graphs, and knot embeddings. As such, the ability to compare and cluster such objects is required in a data analysis pipeline, leading to a need for distances or metrics between them. In this work, we study the interleaving distance on discretizations of these objects, $\mathbb{R}^d$-mapper graphs, where functor representations of the data can be compared by finding pairs of natural transformations between them. However, in many cases, computation of the interleaving distance is NP-hard. For this reason, we take inspiration from the work of Robinson to find quality measures for families of maps that do not rise to the level of a natural transformation, called assignments. We then endow the functor images with the extra structure of a metric space and define a loss function which measures how far an assignment is from making the required diagrams of an interleaving commute. Finally we show that the computation of the loss function is polynomial. We believe this idea is both powerful and translatable, with the potential to be used for approximation and bounds on interleavings in a broad array of contexts.

2.16 Topologically Attributed Graphs

Thomas Needham (Florida State University - Tallahassee, US)

Joint work of Needham; Curry; Mio; Okutan; Russold

I will describe recent work with Curry, Mio, Okutan and Russold which fuses graphical and persistence invariants of datasets. The basic idea is to attribute the nodes of a Reeb or Mapper graph of a dataset with persistence diagrams, which encode localized, higher-dimensional homological features of the data. These enriched graphical summaries can be used, for example, as inputs to a graph neural network for shape classification tasks. I will also discuss the (fairly subtle) theoretical stability properties of these invariants.

2.17 Directed paths and duality

Martin Raussen (Aalborg University, DK)

An important class of Higher Dimensional Automata (HDA) in concurrency theory arises from semaphore protocols or PV-programs originally described by Dijkstra. In order to understand their behaviour, one must analyse the *space* of *all* schedules (directed paths) between (any) start and end state. How can one translate the orders of lock and unlock commands into a recipe describing this space?

By definition, the space of allowed directed paths is an intersection (limit) of elementary spaces – each having the homotopy type of a sphere – in the infinite-dimensional space of all directed paths. There is a homotopy equivalence embedding the (allowed) paths as a *configuration space* into a *finite-dimensional* sphere. The complement of this configuration space in that sphere is a union (colimit) of elementary spaces. Its topology can therefore be described as the homotopy colimit of certain spaces for which we have a “low-dimensional” description arising directly from the PV-encoding. In favourable cases, this homotopy colimit can be described explicitly. Alexander duality allows then to determine the homology of the complement, and hence of the space of all allowed directed paths.

2.18 From Coarse to Fine and Back Again: Geometry and Topology in Machine Learning

Bastian Grossenbacher-Rieck (Helmholtz Zentrum München, DE)

A large driver contributing to the success of deep learning models is their ability to synthesise task-specific features from data. For a long time, the predominant belief was that 'given enough data, all features can be learned.' However, it turns out that certain tasks require imbuing models with inductive biases such as invariances that cannot be readily gleaned from the data! This is particularly true for data sets that model real-world phenomena, creating a crucial need for different approaches. This talk will present novel advances in harnessing multi-scale geometrical and topological characteristics of data. I will particularly focus on how geometry and topology can improve (un)supervised representation learning tasks. Underscoring the generality of a hybrid geometrical-topological perspective, I will furthermore showcase applications from a diverse set of data domains, including point clouds, graphs, and higher-order combinatorial complexes.

2.19 Overview of Discrete Morse Theory

Nick Scoville (Ursinus College - Collegeville, US);
Leonard Wienke (Universität Bremen, DE)

This Overview of Discrete Morse Theory is two-fold.

In the first part, we give an introduction to the basic concepts of Discrete Morse Theory. In particular, we discuss the equivalence of simplicial collapses, acyclic matchings, and poset maps with small fibers. We then define the Morse complex that computes simplicial homology and consider examples.

In the second part, we discuss open problems as well as newer directions of research. We will look at open problems in both random Discrete Morse Theory and the complex of discrete Morse functions. We will then survey several variations of Discrete Morse Theory, inluding stratified and Bestvina-Brady, which may prove useful in simplifying a complex.

2.20 Combinatorial Topological Models for Phylogenetic Reconstruction Networks

Jan F Senge (Universität Bremen, DE, Polish Academy of Science, PL)

Joint work of Dłotko, Pawel; Senge, Jan; Stefanou, Anastasios
Main reference Dłotko Paweł, Senge JF, Stefanou Anastasios (2023) Combinatorial Topological Models for Phylogenetic Networks and the Mergegram Invariant. arXiv preprint arXiv:2305.04860.
URL https://arxiv.org/abs/2305.04860

Phylogenetic networks are vital for understanding complex evolutionary processes, where traditional tree-like structures fall short. The application of topological data analysis (TDA) has emerged as a powerful approach for exploring such networks, revealing underlying geometric and topological structures. This talk focuses on a lattice theoretical approach of representing such networks and relating them to TDA. We will discuss the applications of TDA techniques in analyzing phylogenetic networks, aiming to uncover hidden patterns and gain deeper insights into their evolutionary dynamics. Additionally, we introduce the facegram, a simplicial lattice model that generalizes the dendrogram model for phylogenetic trees, which enables an alternative way to visualize filtrations of complexes, and show some more recent applications of these ideas and connections.

2.21 Reeb Graphs and Their Variants: Theory and Applications

Bei Wang Phillips (University of Utah - Salt Lake City, US)

A Reeb graph is a graphical representation of a scalar function on a topological space that encodes the topology of the level sets. Reeb graphs and their variants are popular tools in topological data analysis and visualization. As an overview talk for TDA+statistics, I will review theoretical advances in studying Reeb graphs and their variants, as well as their applications in data mining and machine learning.

2.22 Persistent cup modules

Ling Zhou (Ohio State University - Columbus, US)

One-dimensional persistent homology is arguably the most important and heavily used computational tool in topological data analysis. Additional information can be extracted from datasets by studying multi-dimensional persistence modules and by utilizing cohomological ideas, e.g. the cohomological cup product. In this work, given a single parameter filtration, we investigate a certain 2-dimensional persistence module structure associated with persistent cohomology, where one parameter is the cup-length and the other is the filtration parameter. This new persistence structure, called the persistent cup module, is induced by the cohomological cup product and adapted to the persistence setting. Furthermore, we show that this persistence structure is stable. By fixing the cup-length parameter, we obtain a 1-dimensional persistence module and again show it is stable in the interleaving distance sense, and study their associated generalized persistence diagrams. This is a joint work with F. Mémoli and A. Stefanou.